Laser Physics 18.

Pulsed lasers

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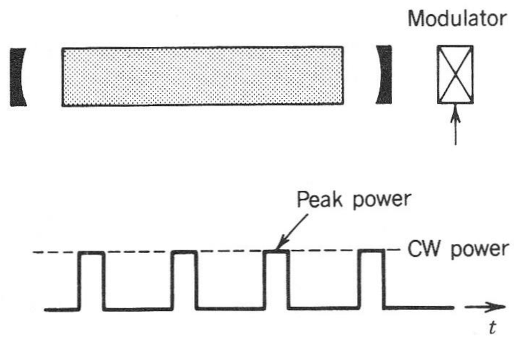
Pulsed lasers

Pulsed laser operation can be obtained from a cw laser by using an external switch or modulator, e.g. a rotating disc with holes (chopper). Advantage: signal/noise ratio increases (e.g. blocking fluctuating background when measuring small signal amplitudes).

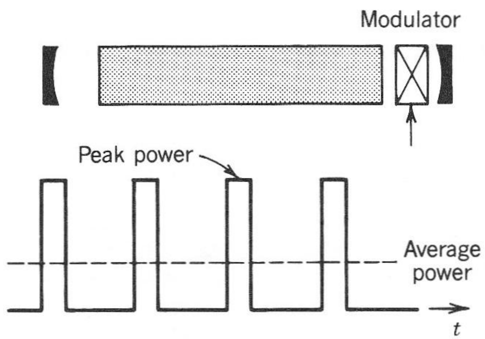
Disadvantage of the external modulation:

- power loss,
- peak power is the same.



Advantage of an internal modulator: short, very high peak power pulses.

Possibilities: gain or loss can be modulated!

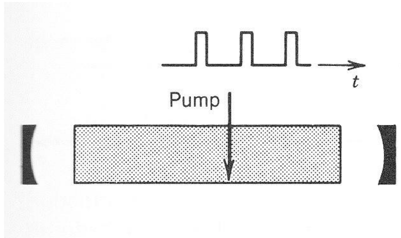


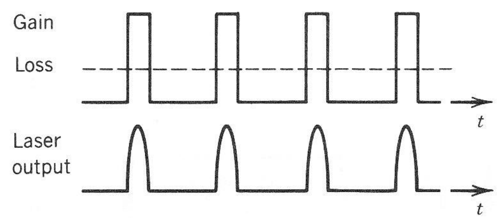
Remember conditions of laser operation: gain condition!

Pulsed lasers

Methods: **gain switching, *Q* – switching** (or cavity damping) and mode locking.

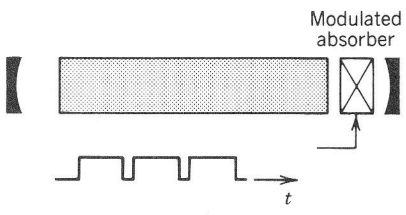
Gain switching

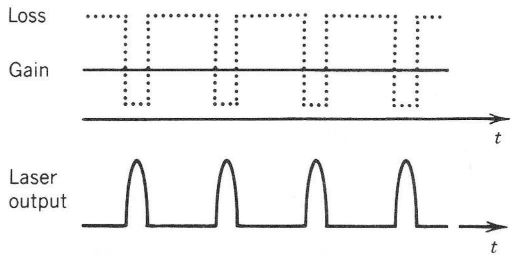




E.g. flash lamp pumping (solid state lasers), electric current modulation (semiconductor laser), most frequently used.

Q- switching





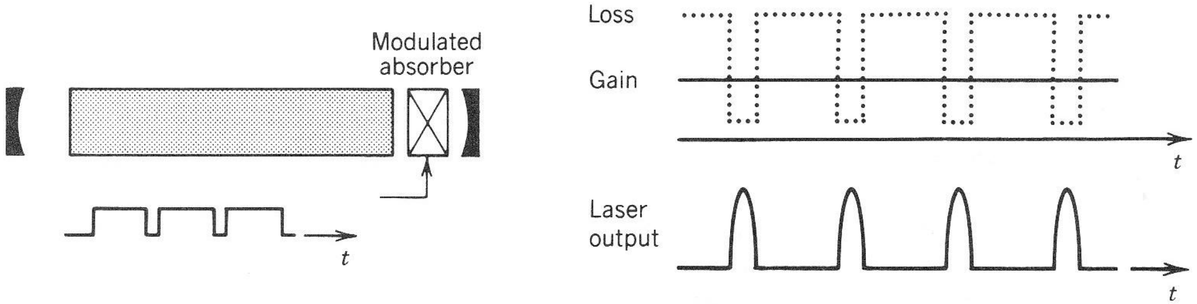
During cw pumping energy is stored in the material in form of population difference – suitable laser material is needed!

$$Q \underset{\text{def}}{\equiv} \frac{2\pi \cdot \text{stored energy}}{\text{energy loss per cycle}} = \frac{2\pi \cdot \nu}{\alpha_r c} = \frac{\nu}{\Delta \nu_r} = 2\pi\tau_r\nu.$$

Pulsed lasers

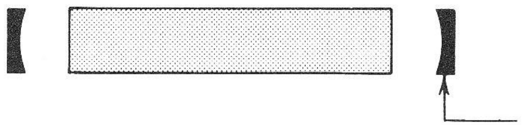
Methods: gain switching, ***Q* –switching (or cavity damping)** and mode locking.

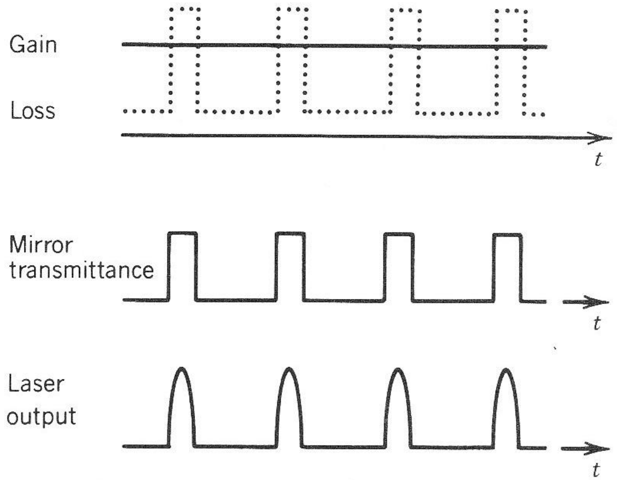
Q-switching



In both cases modulation of the loss, but in opposite way.

Cavity damping





Based on storing photons instead of population difference during the off-time and releasing them during the on-times!

Mode locking – differs significantly from other techniques, the phase locking of the modes provides **extremely short pulses (fs).**

Pulsed lasers

Analyses of transient effects – rate equation for the photon-number density and for the inversion density

$$\frac{dn}{dt} = -\frac{n}{\tau_r} + NW_i,$$

$$W_i = \Phi\sigma(\nu) = nc\sigma(\nu),$$

$$N_t = \frac{\alpha_r}{\sigma(\nu)} = \frac{1}{c\tau_r\sigma(\nu)} \rightarrow \sigma(\nu) = \frac{1}{c\tau_r N_t},$$

$$W_i = nc\frac{1}{c\tau_r N_t} = \frac{n}{\tau_r N_t}$$

$$\boxed{\frac{dn}{dt} = -\frac{n}{\tau_r} + \frac{N}{N_t}\cdot\frac{n}{\tau_r}}.$$

Depends on the resonator lifetime and the net photon gain arising from stimulated processes (the spontaneous emission is assumed to be small)

$N = N_2 - N_1$ population difference in unit volume!

If $N > N_t$, then $dn\,/\,dt > 0$ and n increases! In steady state ($dn\,/\,dt = 0$) $N = N_t$.

Pulsed lasers

Analyses of transient effects – rate equation for the photon number density and for the population difference (3-level pumping scheme)

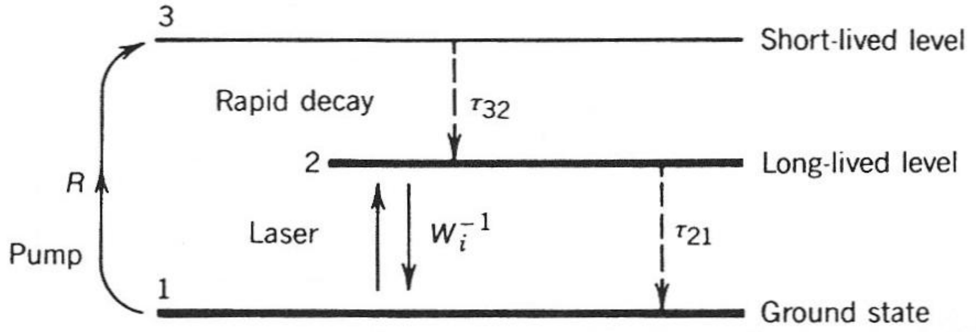
Depends on the pumping configuration. In case of 3-level pumping scheme, N_3 can be neglected because of the fast decay of level 3. If N_a is the total atomic number density and $t_{sp} = \tau_{21}$ (there is no non-radiative decay):

$$N_a = N_1 + N_2,$$

$$N = -N_1 + N_2,$$

$$\frac{N_a + N}{2} = N_2, \quad \frac{N_a - N}{2} = N_1$$

$$\frac{dN_2}{dt} = R - \frac{N_2}{t_{sp}} - W_i (N_2 - N_1),$$



$$\frac{1}{2}\frac{dN}{dt} = R - \frac{N_a}{2t_{sp}} - \frac{N}{2t_{sp}} - W_i N \quad \rightarrow \quad \frac{dN}{dt} = 2R - \frac{N_a}{t_{sp}} - \frac{N}{t_{sp}} - 2\frac{N}{N_t}\frac{n}{\tau_r},$$

$$W_i = \frac{n}{\tau_r N_t}$$

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Analyses of transient effects – rate equation for the photon number density and for the population difference (3-level pumping scheme)

$$\frac{dN}{dt} = 2R - \frac{N_a}{t_{sp}} - \frac{N}{t_{sp}} - 2\frac{N}{N_t}\frac{n}{\tau_r}$$

In steady-state in the absence of amplifier radiation the inversion density is equal to small-signal population difference, N_0:

$$\frac{dN}{dt} = 0 \qquad n = 0, \quad N = N_0.$$

$$0 = 2R - \frac{N_a}{t_{sp}} - \frac{N_0}{t_{sp}} \quad \rightarrow \quad N_0 = 2R\,t_{sp} - N_a$$

$$\boxed{\frac{dN}{dt} = \frac{N_0}{t_{sp}} - \frac{N}{t_{sp}} - 2\frac{N}{N_t}\frac{n}{\tau_r}} \qquad \boxed{\frac{dn}{dt} = -\frac{n}{\tau_r} + \frac{N}{N_t}\cdot\frac{n}{\tau_r}}$$

2 coupled nonlinear differential equations!

Factor of 2 because of the 3-level pumping scheme!

Pulsed lasers

Analyses of transient effects – rate equation for the photon number density and for the population difference (3-level pumping scheme)

In steady-state:

$$\frac{dN}{dt}=0, \quad \frac{dn}{dt}=0 \quad \rightarrow \quad N=N_t$$

$$0=\frac{N_0}{t_{sp}}-\frac{N_t}{t_{sp}}-2\frac{N_t}{N_t}\frac{n}{\tau_r} \quad \rightarrow \quad n=\left(N_0-N_t\right)\frac{\tau_r}{2t_{sp}}.$$

Steady-state photon number density of 3-level system

Pulsed lasers

Gain switching

Switching of the pumping rate R is equivalent to modulate $N_0 = 2R\,t_{sp} - N_a$. The small-signal population difference changes from N_{0a} to N_{0b}! Time evolution of $N(t)$ and $n(t)$:

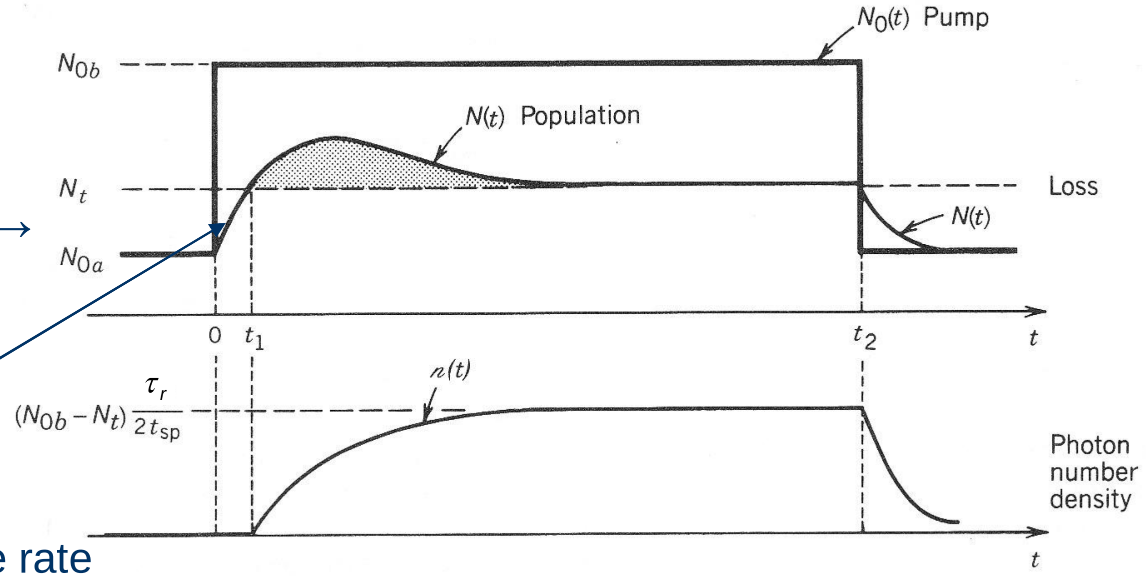
$t < 0$, $N(t) = N_{0a} < N_t$, below the threshold there is no oscillation

$t = 0$ the pump is turned on, $N_{0a} \rightarrow N_{0b}$, $N(t)$ increases, as long as $N(t) < N_t$, $n(t) = 0$

$$\frac{dN}{dt} = \frac{N_0}{t_{sp}} - \frac{N}{t_{sp}} - 2\frac{N}{N_t}\frac{n}{\tau_r} = \frac{N_0 - N}{t_{sp}}$$

$t = t_1$ $N(t) = N_t$, $n(t)$ increases, the rate of increase of $N(t)$ slows, then $N(t)$ begins to decay because of the coupled equations. Finally $N(t) = N_t$.

$t = t_2$ the pump is turned off, $N_{0b} \rightarrow N_{0a}$, $N(t)$ and $n(t)$ decrease.



$$\frac{dN}{dt} = \frac{N_0}{t_{sp}} - \frac{N}{t_{sp}} - 2\frac{N}{N_t}\frac{n}{\tau_r}, \quad \frac{dn}{dt} = -\frac{n}{\tau_r} + \frac{N}{N_t}\cdot\frac{n}{\tau_r}$$

Actual shape of $n(t)$ is obtained by numerical solution, depends on t_{sp}, τ_r, N_t, N_{0a} and N_{0b}.

Pulsed lasers

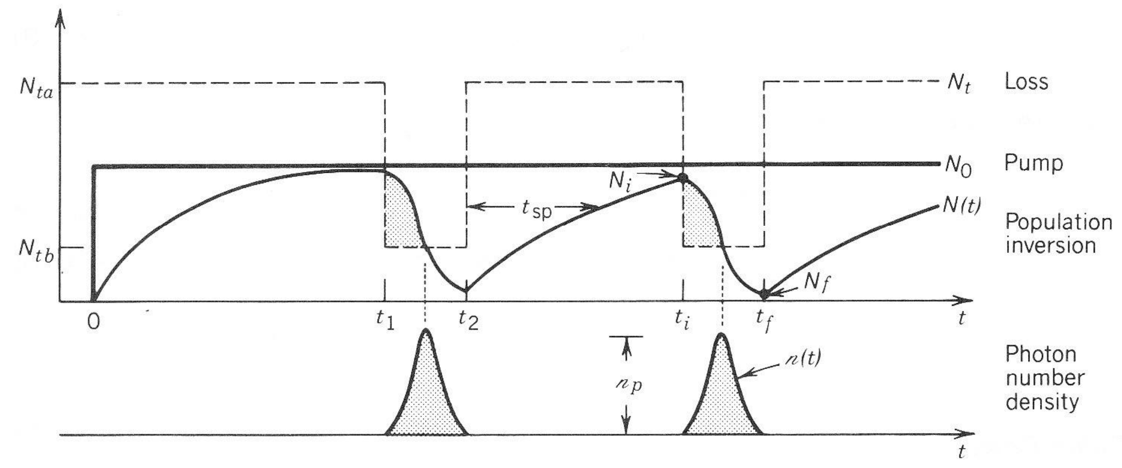
Q-switching

Switching the resonator loss coefficient α_r, because of $N_t = \alpha_r / \sigma(\nu)$ this means the modulation of N_t between N_{tb} and N_{ta}. The small-signal inversion density $N_0(t)$ remains fixed! Evolution of $N(t)$, $n(t)$:

$t = 0$ the pump is turned on, N_0 follows a step function, $N_t = N_{ta} > N_0$, no lasing, $N(t)$ builds up.

$t = t_1$ the loss is decreased, $N_t = N_{tb} < N_0$, starts the oscillation and the increase of $n(t)$, $N(t)$ decreases. When $N(t) < N_{tb}$, $n(t)$ quickly decreases (time constant ~ τ_r).

$t = t_2$ the loss is reinstated, $N_t = N_{ta}$, $N(t)$ again increases.



$$\frac{dN}{dt} = \frac{N_0}{t_{sp}} - \frac{N}{t_{sp}} - 2\frac{N}{N_t}\frac{n}{\tau_r}, \quad \frac{dn}{dt} = -\frac{n}{\tau_r} + \frac{N}{N_t}\cdot\frac{n}{\tau_r}$$

Pulsed lasers

Q-switching (cont.)

The pulse length depends on the resonator lifetime τ_r:

$$\tau_r = \frac{1}{\alpha_r c} = \frac{2L}{c} \frac{1}{-\ln R_1 R_2}$$

A high loss and short length resonator is advantageous to use for Q-switching (e.g. with low reflectivity mirrors).

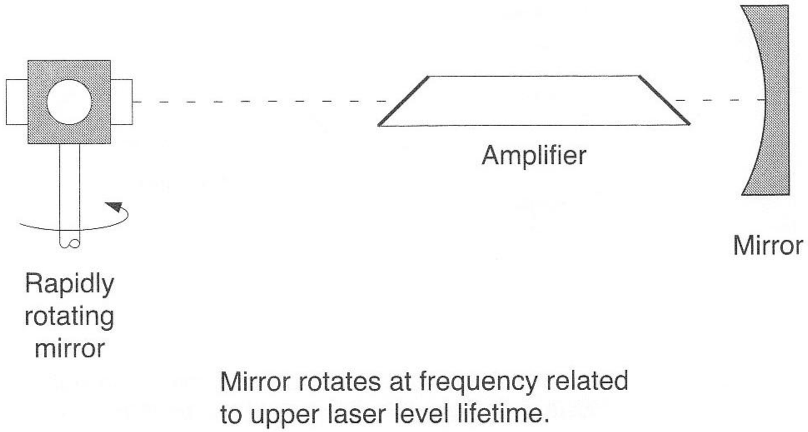
The upper level lifetime must be in the range of ~*ms*.

Typical pulse length: in the range of *ns* – 10 *ns*,
peak power: *MW* – 10 *MW*

Pulsed lasers

Q-switching methods

1. Mechanical method



First realization, the rotating speed depends on t_{sp}.

E.g. rubin laser $\tau_{21} \sim t_{sp} \sim 3\ ms$

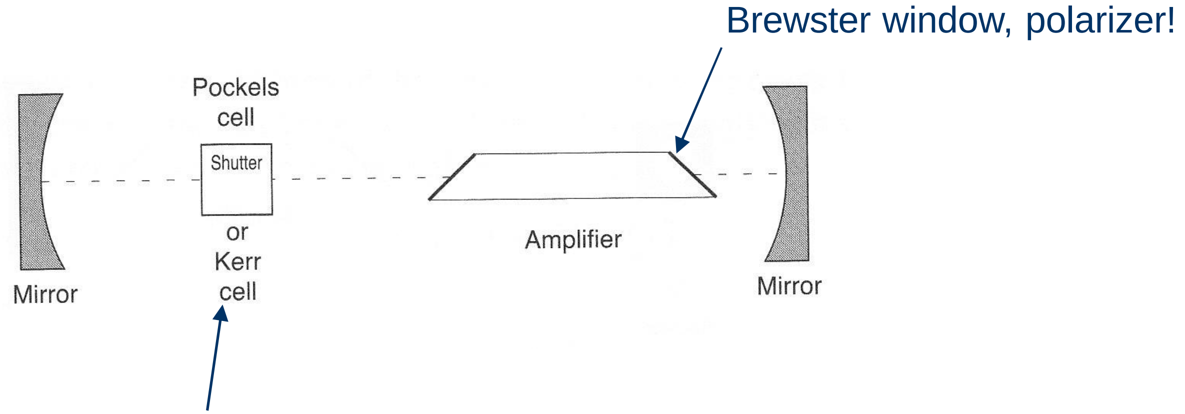
With the movement of one reflecting surface: $T_{per} = 5\ ms \rightarrow 200$ revolutions/sec, 12000 revolutions/min, very high rotation speed. Attenuation of vibration and the precise alignment are the difficulties!

Pulsed lasers

Q-switching methods (cont.)

2. Electrical solutions – electro-optic shutter

   Basis: electro-optic effect. The shutter is an electro-optic crystal that becomes birefringent when electrical field is applied across the crystal, the refractive index or the phase shift can be tuned with the electric field.



When the voltage is on the shutter rotates the plane of polarization by 45°.
2x passes rotates by 90°, oscillation is blocked by the polarizer.

Pulsed lasers

Q-switching methods (cont.)

2. Electrical solutions – electro-optic shutter (cont.)

   Pockels-cell – nonlinear crystal in which an applied dc (5-10 kV) voltage induces a change in the crystal's refractive index.

   $\left.\begin{array}{c} \text{KDP } (KH_2PO_4) \\ LiNbO_3 \end{array}\right\}$ visible and near-infrared

   CdTe — middle-infrared

   Kerr-cell – normally isotropic liquid becomes birefringent by aligning the molecules with electric field

   e.g. liquid nitrobenzene ($C_6H_5NO_2$), disadvantage – toxic and >10 kV high voltage is necessary!

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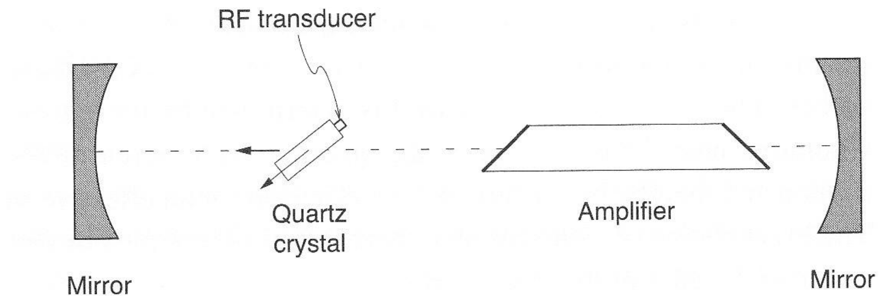
Q-switching methods (cont.)

2. Electrical solutions – acousto-optic shutter
   acousto-optic crystal in the resonator, e.g. a quartz crystal



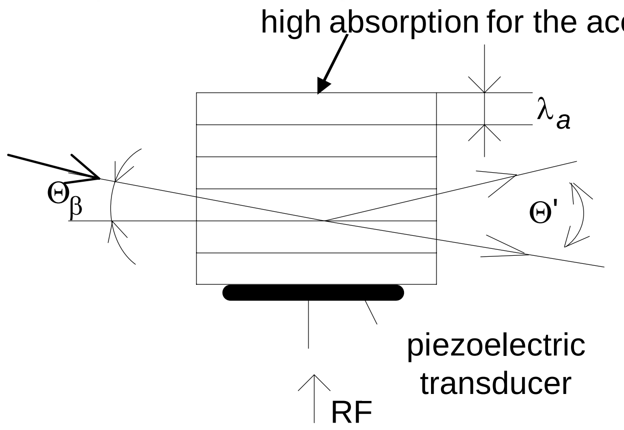
Traveling acoustic waves in the crystal. The acoustic wavelength is

$$\lambda_a = v / \nu_a,$$

v is the velocity of sound, ν_a is the frequency of the RF field.

The light is diffracted on the traveling wave (eff. can be 50%, oscillation stops because of the high loss).

$$\Theta_\beta = \frac{\lambda}{2\lambda_a}$$



$$\Theta' = \frac{\lambda}{\lambda_a}$$

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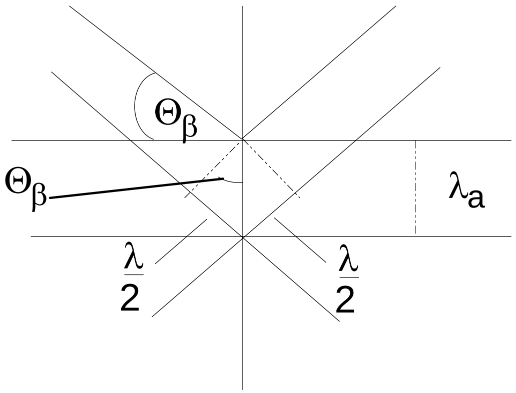
Q-switching methods (cont.)

2. Electrical solutions – acousto-optic shutter (cont.)

   Diffraction only in definite angles according to the Bragg condition (like in crystals the x-ray diffraction)



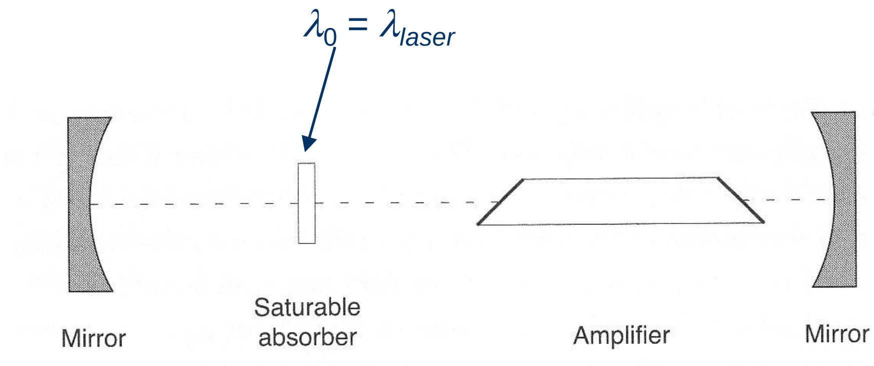
$$\Theta_\beta \cong \sin\Theta_\beta = \frac{\lambda_0}{2\lambda_a}$$

After switching off the RF field the diffraction disappears and the laser oscillation starts. Repeated Q-switching is possible.

Pulsed lasers

Q-switching methods (cont.)

3. Passive Q-switching



The saturable absorber causes high loss at the beginning. With increasing flux density the absorber saturates and becomes transparent.

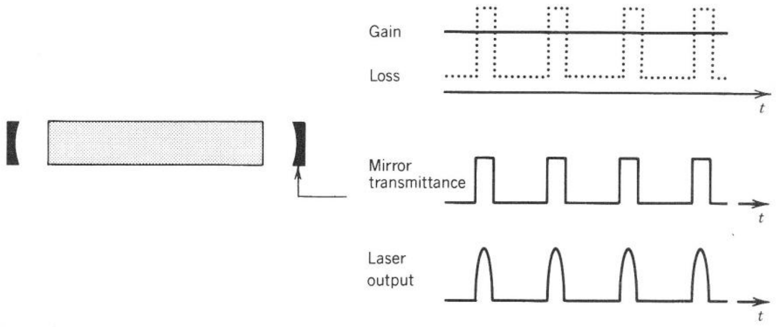
Suitable materials: BDN dye (4-dimetil-aminodithiobenzil-nikkel) in ethanol
for 1.06 μm (Nd:YAG laser)
SF_6 gas for 10.6 μm (CO_2 laser)
semiconductor layer on mirror (SESAM – semiconductor saturable absorber mirror)

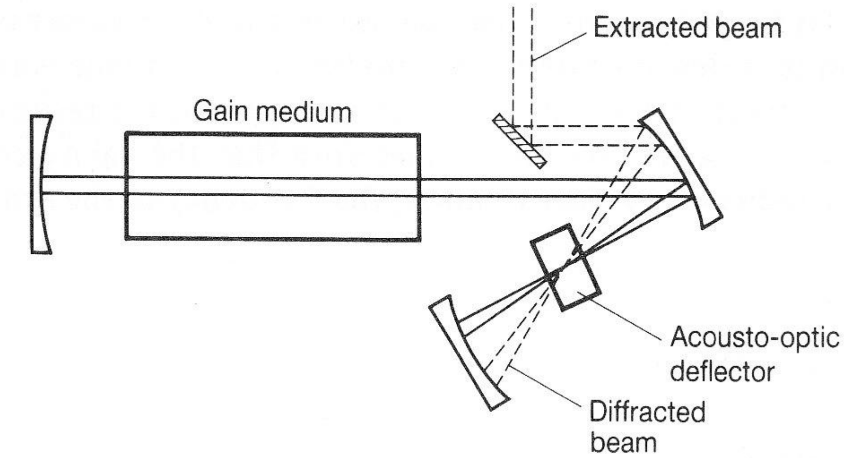
Important: saturation flux density must be low
saturation time must be low (~ μs)

Pulsed lasers

<u>Method of cavity damping</u>

Instead of the modulation of the mirror transmittance an acousto-optic deflector can be used:

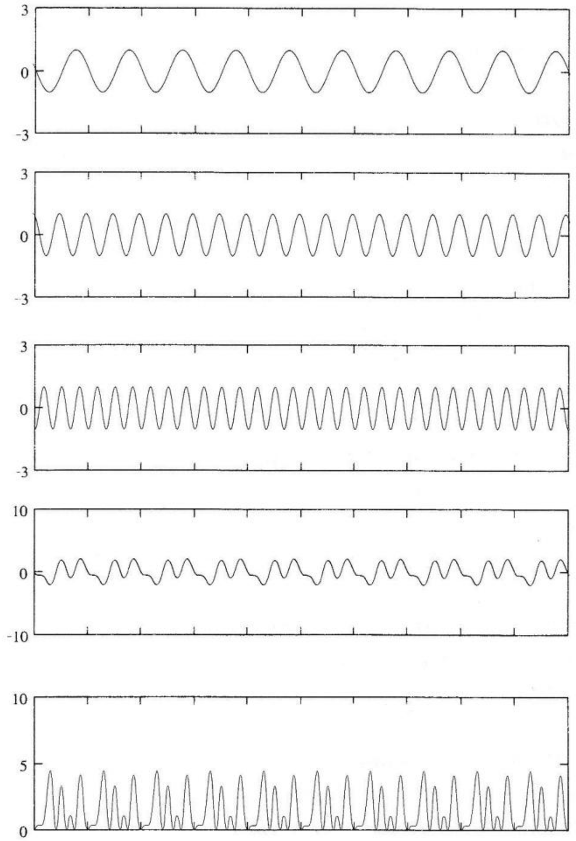




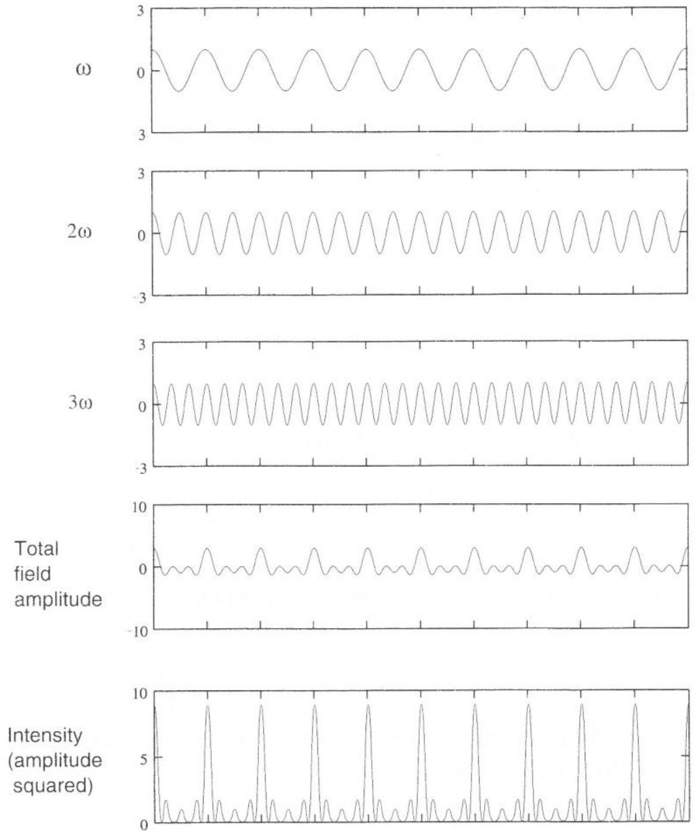
Pulsed lasers

Mode locking

Ultra short pulse generation by the synchronization of phases when large number of longitudinal modes oscillate. Without synchronization random fluctuating output:



Sum of three out-of-phase waves



Sum of three waves in phase

Pulsed lasers

Mode locking – theory

The sum of M longitudinal modes with ϕ fixed phase and with equal frequency difference $\Delta\nu_n$ (large operational bandwidth is necessary, e.g. dye or solid-state laser):

$$E(t) = \sum_{k=-M/2}^{M/2} E_k \cdot e^{i2\pi(\nu_0 + k\Delta\nu_n)t + i\phi}, \quad \Delta\nu_n = \frac{c}{2L}$$

Simplification – equal amplitudes: $E_k = E_0$, sum of geometric series:

$$E(t) = E_0 \cdot e^{i2\pi\nu_0 t + i\phi} \sum_{k=-M/2}^{M/2} \cdot e^{i2\pi k\Delta\nu_n t} = E_0 \cdot \frac{\sin(M\pi \cdot t \cdot \Delta\nu_n)}{\sin(\pi \cdot t \cdot \Delta\nu_n)} \cdot e^{i2\pi\nu_0 t + i\phi}$$

$$I(t) = |E_0|^2 \cdot \frac{\sin^2(M\pi \cdot t \cdot \Delta\nu_n)}{\sin^2(\pi \cdot t \cdot \Delta\nu_n)}$$

Pulsed lasers

Mode locking – theory (cont.)

$$I(t) = |E_0|^2 \cdot \frac{\sin^2(M\pi \cdot t \cdot \Delta\nu_n)}{\sin^2(\pi \cdot t \cdot \Delta\nu_n)}, \quad \Delta\nu_n = \frac{c}{2L}$$

Pulse maxima occur when $\pi \cdot t \cdot \Delta\nu_n = 0, \pi, 2\pi, \ldots q\pi$, q is integer (the denominator is 0):

$$I(t)_{lim} = \lim_{\pi \cdot t \cdot \Delta\nu_n \to 0} |E_0|^2 \cdot \frac{\sin^2(M\pi \cdot t \cdot \Delta\nu_n)}{\sin^2(\pi \cdot t \cdot \Delta\nu_n)} = \lim_{\pi \cdot t \cdot \Delta\nu_n \to 0} |E_0|^2 \cdot \frac{M^2(\pi \cdot t \cdot \Delta\nu_n)^2}{(\pi \cdot t \cdot \Delta\nu_n)^2} = M^2 |E_0|^2$$

$$\boxed{I(t)_{max} = M^2 |E_0|^2}.$$

Maximum intensity is *M* times the average intensity!!!

Distance of maxima: $\pi \cdot t \cdot \Delta\nu_n = \pi \rightarrow$

$$\boxed{t_{sep} = \frac{1}{\Delta\nu_n} = \frac{2L}{c}}$$

Running time!!

Minimum: $M\ \pi \cdot t \cdot \Delta\nu_n = \pi \rightarrow$

$$\boxed{t_p \cong \frac{1}{M\Delta\nu_n} = \frac{1}{B}}$$

~ inversely depends on the operational bandwidth!!

Pulse length: ~ the distance of the first minimum from the maximum place

Pulsed lasers

Mode locking (cont.)

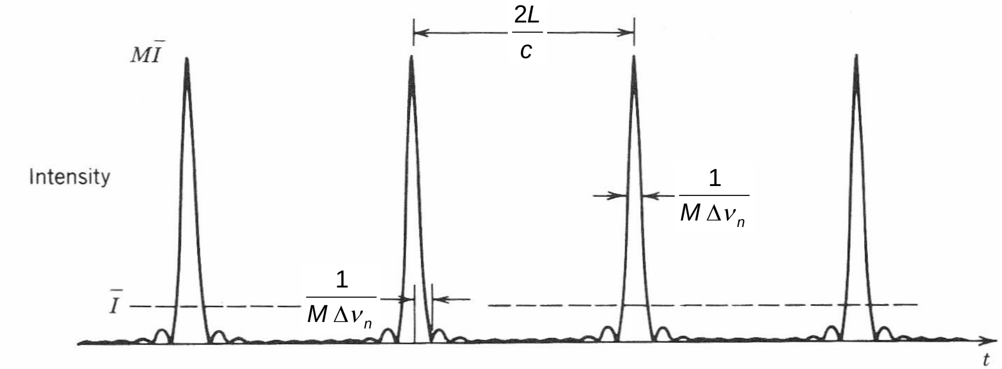
The intensity is a periodic function in time, its characteristics:

- **periodic maxima according to the running time in the resonator ($2L / c$ in time and $2L$ distance in space)**
- **Pulse length inversely proportional to the operational bandwidth of the amplifier**
- **peak intensity = average intensity x the number of coupled modes**

$$I(t) = |E_0|^2 \cdot \frac{\sin^2(M\pi \cdot t \cdot \Delta\nu_n)}{\sin^2(\pi \cdot t \cdot \Delta\nu_n)}$$

$$t_p \cong \frac{1}{M\Delta\nu_n} = \frac{1}{B}$$

in time →

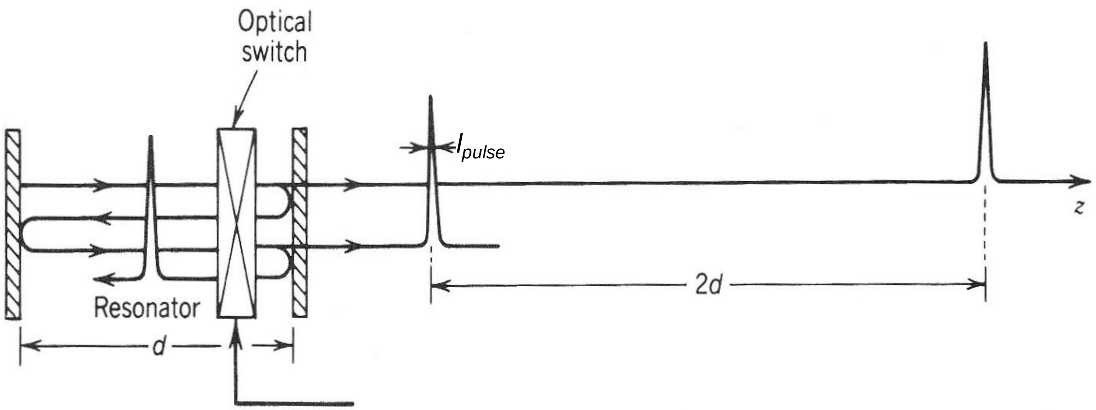


$$B = 100\,THz \quad \rightarrow \quad t_p \sim 10^{-14} s = 10 fs$$

Pulsed lasers

Mode locking (cont.)

in space →

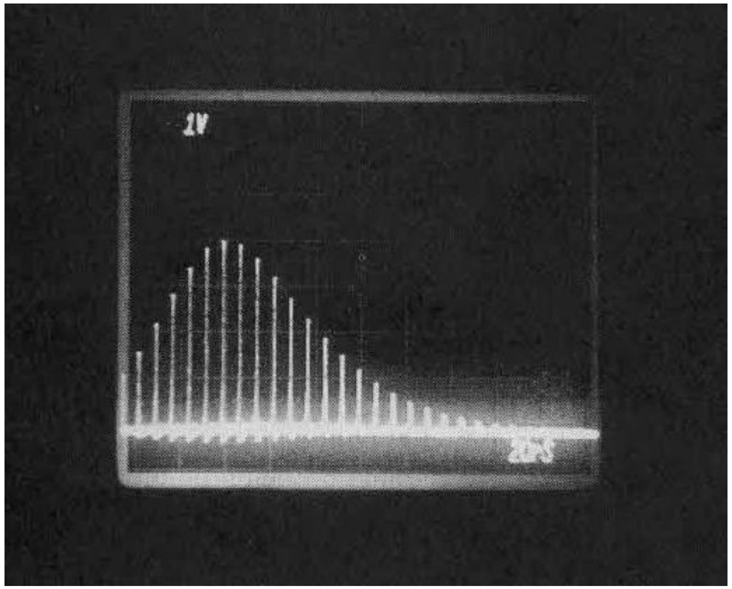


In the reality the amplitudes are different, their form depend on the shape of the gain coefficient.

$$t_p \cong \frac{\beta}{M \Delta \nu_n},$$

β~1, depends on the shape of the gain coefficient! For inhomogeneous amplifier: β=0.441.

~200 ps pulse train of a mode locked Nd:YAG laser.



Pulsed lasers

<u>Mode locking</u> – practical methods

- Active mode locking, external power source is used for the control of the switch: electro-optic or acousto-optic cell in the resonator (like for *Q*-switching), or synchronized pumping (pumping pulses with frequency $c / 2L$, difficult to realize)
- Passive mode locking, no external power source, a saturable absorber or material with nonlinear variation of the refractive index is applied most frequently close to one mirror. Pulse with sufficiently large irradiance saturates the absorber, therefore goes through without loss. The recovery time of the absorber must be shorter than the roundtrip time in the resonator!