Laser Physics 14.

Coherent optical amplifier (cont.)

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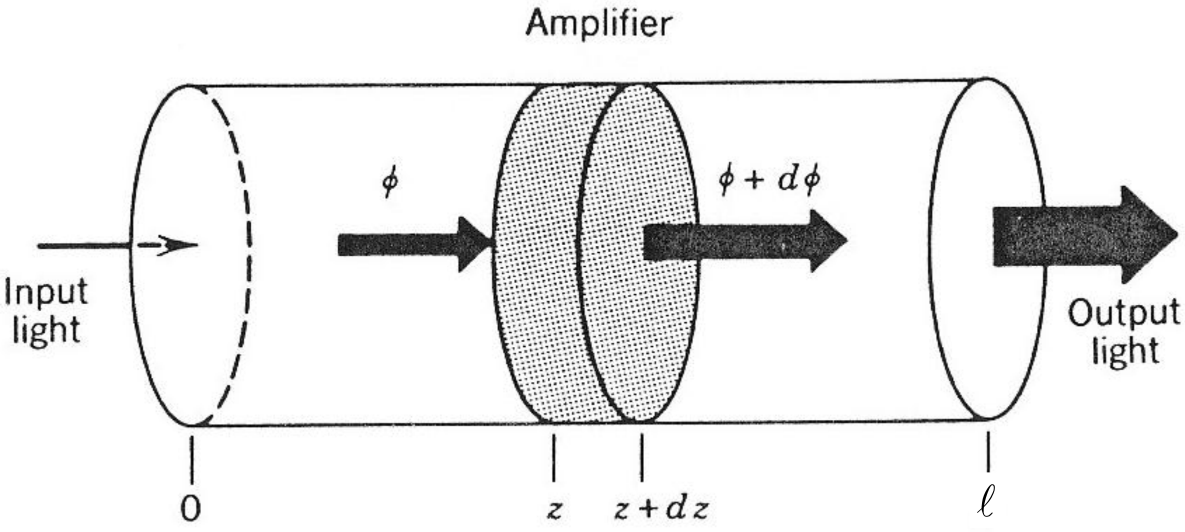
Coherent optical amplifier

Gain, bandwidth, phase shift, power source, nonlinearity and noise

The monochromatic optical plane wave traveling in *z* direction in the laser material with frequency ν can be characterized:

$$\mathrm{Re}\left\{E(z)e^{j2\pi\nu t}\right\},\quad I(z)=\frac{E^2(z)}{2\eta},\quad \Phi(z)=\frac{I(z)}{h\nu},$$

η Is the vacuum impedance.



Coherent optical amplifier

Amplifier nonlinearity or gain saturation

Rev..: $\gamma(\nu) = N\sigma(\nu) = N\dfrac{\lambda^2}{8\pi t_{sp}} g(\nu),\quad N = \dfrac{N_0}{1 + W_i\tau_s},\qquad \tau_s = \tau_2 + \tau_1\left(1 - \dfrac{\tau_2}{\tau_{21}}\right).$

gain coefficient

Φ-dependence $\qquad W_i = \Phi\sigma(\nu).$

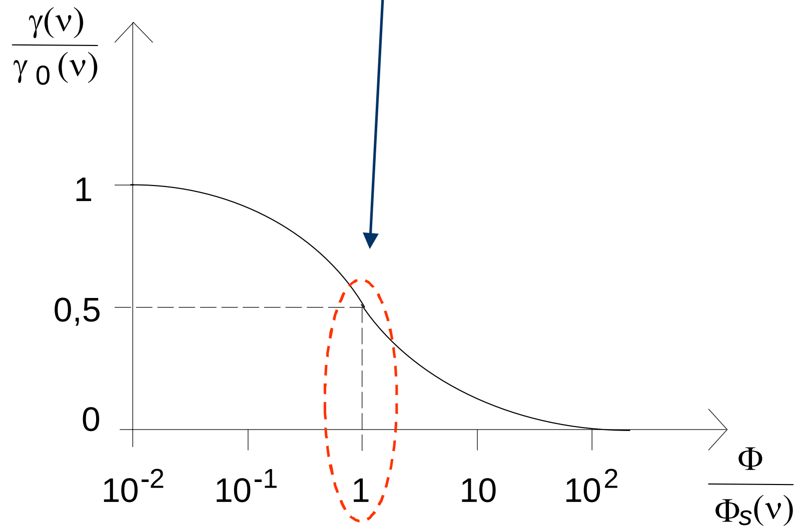
$$N = \frac{N_0}{1 + \Phi/\Phi_s(\nu)},\qquad \frac{1}{\Phi_s(\nu)} = \tau_s\sigma(\nu) = \frac{\lambda^2}{8\pi}\frac{\tau_s}{t_{sp}} g(\nu),$$

Φ_s saturation photon-flux density.

$$\gamma(\nu) = \frac{\gamma_0(\nu)}{1 + \Phi/\Phi_s(\nu)},$$

$$\gamma_0(\nu) = N_0\sigma(\nu) = N_0\frac{\lambda^2}{8\pi t_{sp}} g(\nu).$$

small signal gain

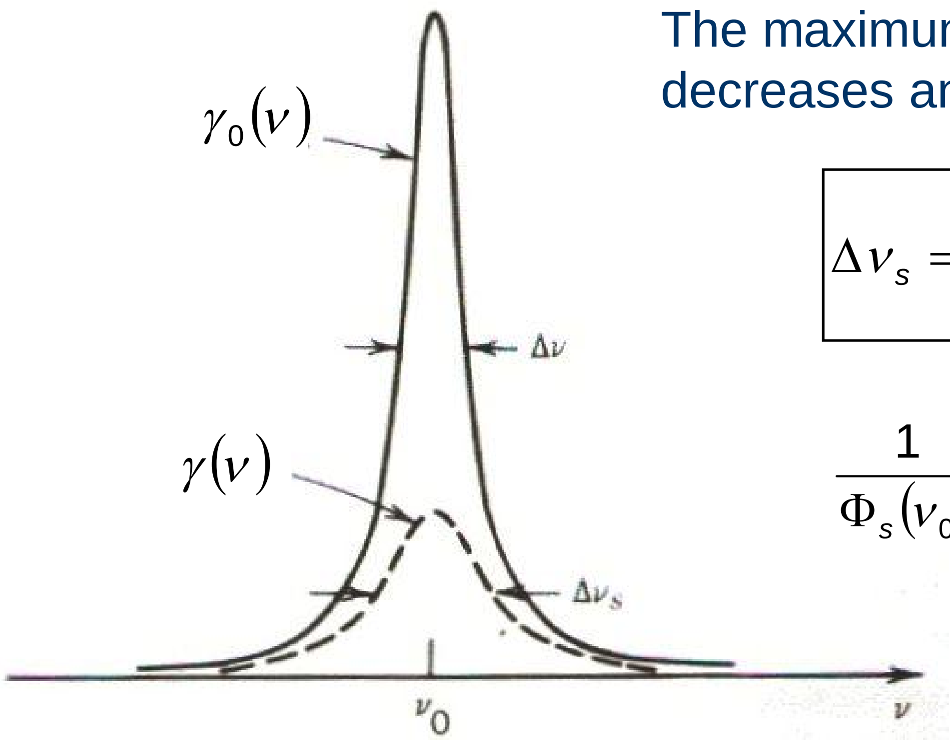


Coherent optical amplifier

Amplifier nonlinearity or gain saturation – frequency dependence

Depends on the broadening behavior of the medium, different for homogeneous and inhomogeneous media.

Homogeneously broadened medium



The maximum of the Lorentz curve decreases and the FWHM increases:

$$\boxed{\Delta\nu_s = \Delta\nu\left[1+\frac{\Phi}{\Phi_s(\nu_0)}\right]^{1/2}}$$

$$\frac{1}{\Phi_s(\nu_0)} = \tau_s S g(\nu_0) = \tau_s S \frac{2}{\pi\Delta\nu}$$

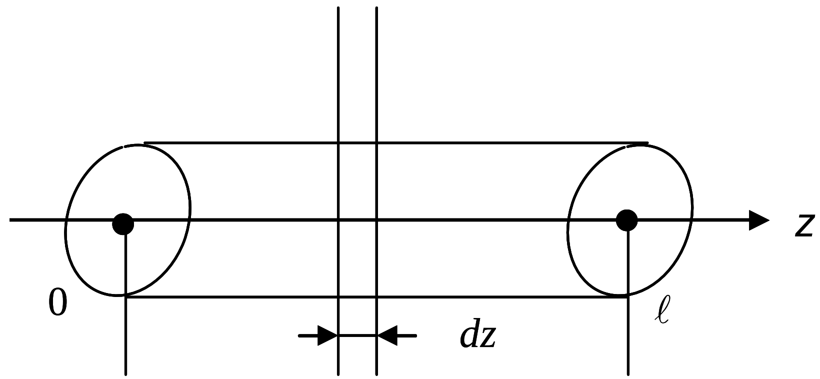
Coherent optical amplifier

<u>Amplifier nonlinearity or gain saturation</u>

Homogeneously broadened medium (cont.)

Gain of a homogeneous medium of length ℓ? $\quad G = \dfrac{\Phi(\ell)}{\Phi(0)} = ?$

$\Phi(z)$ is the photon-flux density at z



$$\frac{d\Phi}{dz} = NW_i = \underbrace{N\sigma}_{\gamma}\Phi,$$

$$\frac{d\Phi}{dz} = \frac{\gamma_0 \Phi}{1+\dfrac{\Phi}{\Phi_s}},$$

$$\left(\frac{1}{\Phi}+\frac{1}{\Phi_s}\right)d\Phi = \gamma_0 dz \xrightarrow{\int_0^\ell} ln\left[\frac{\Phi(\ell)}{\Phi(0)}\frac{\Phi_s}{\Phi_s}\right] + \frac{\Phi(\ell)-\Phi(0)}{\Phi_s} = \gamma_0 \ell$$

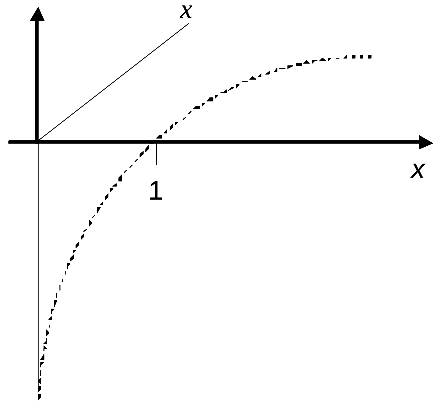
$$\boxed{ln(Y)+Y = [ln(X)+X]+\gamma_0\ell, \quad X = \Phi(0)/\Phi_s\,, \quad Y = \Phi(\ell)/\Phi_s\,.} \qquad G = \frac{\Phi(\ell)}{\Phi(0)} = \frac{Y}{X} = ?$$

There are analytic solutions only in two limiting cases!

Coherent optical amplifier

<u>Amplifier nonlinearity or gain saturation</u>

Homogeneously broadened medium (cont.)



1. X and $Y \ll 1$, the photon-flux densities are much smaller than the saturation photon-flux density

$$ln(Y) \approx ln(X) + \gamma_0 \ell, \quad \rightarrow \quad ln\left(\frac{Y}{X}\right) = \gamma_0 \ell, \quad \rightarrow \quad Y \approx X\, exp(\gamma_0 \ell), \quad \rightarrow \quad G = exp(\gamma_0 \ell).$$

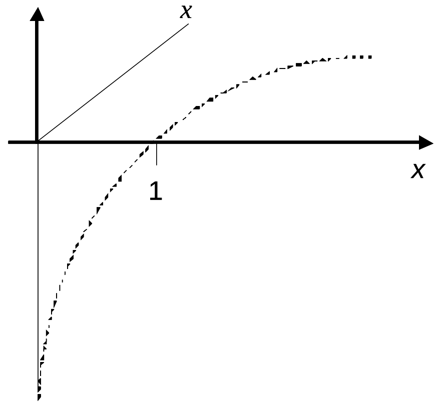
X and *Y* are negligible in comparison with *ln X* and *ln Y*. There is linear dependence between the input and output signals for a given length of the medium, the gain depends on γ_0, this is the origin of the name **small signal gain**!

Coherent optical amplifier

Amplifier nonlinearity or gain saturation

Homogeneously broadened medium (cont.)

2. X *és* $Y >> 1$, the photon-flux densities are much higher than the saturation photon-flux density



$$Y \approx X + \gamma_0 \ell, \quad \Phi(\ell) \approx \Phi(0) + \gamma_0 \Phi_s \ell \approx \Phi(0) + \frac{N_0 \ell}{\tau_s}.$$

$$G = \frac{Y}{X} \cong 1 + \gamma_0 \ell \frac{1}{X} = 1 + \gamma_0 \ell \frac{\Phi_s}{\Phi(0)} \approx 1.$$

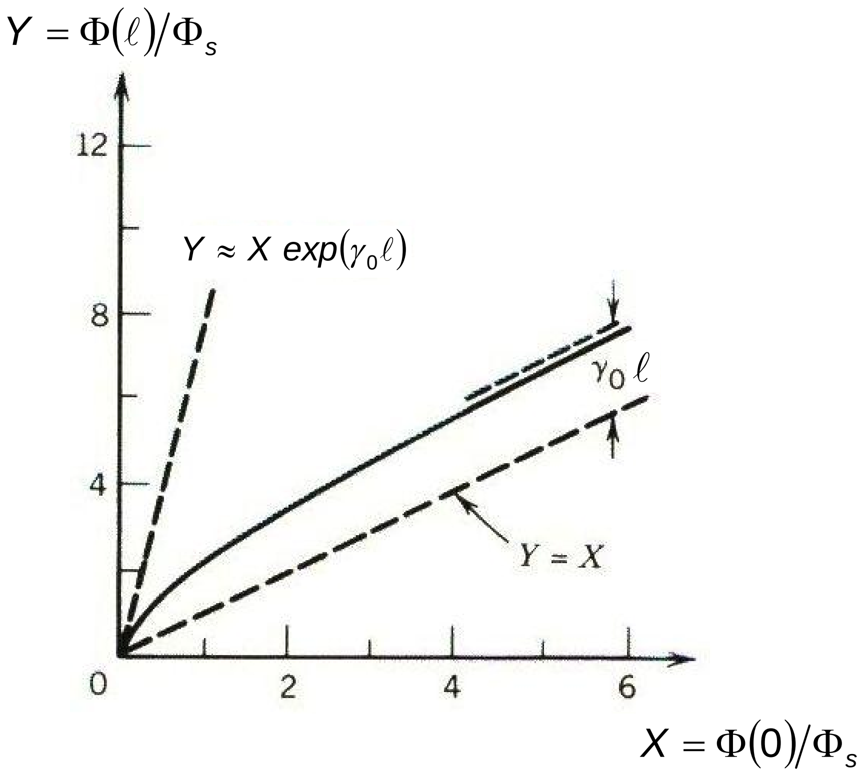
ln X and *ln Y* can be neglected in comparison with *X* and *Y.* Under such heavily saturated conditions there is only a constant grow in the output that is independent from the input photon-flux density. The medium becomes almost transparent!

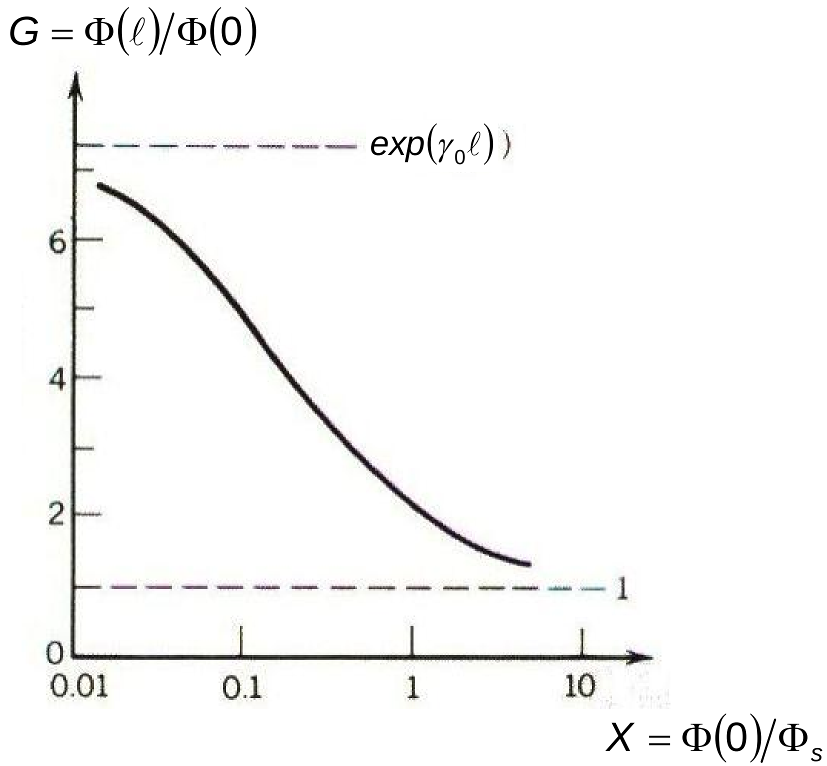
Coherent optical amplifier

Amplifier nonlinearity or gain saturation

Homogeneously broadened medium (cont.)

For intermediate values of *X* and *Y* there are numerical solutions:



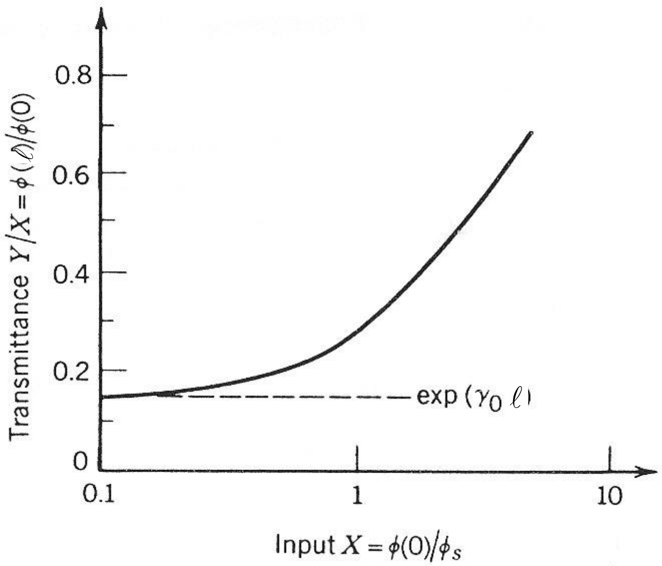


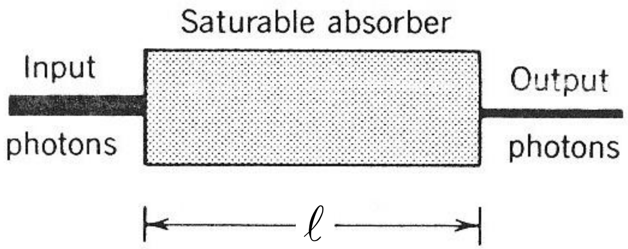
Coherent optical amplifier

Amplifier nonlinearity or gain saturation

Homogeneous saturable absorber

The gain coefficient is negative when the population is normal than inverted (in thermal equilibrium), that is $N_0 < 0$, the medium provides attenuation than amplification. The attenuation coefficient $\alpha(\nu) = -\gamma(\nu)$ also suffers from saturation with growing photon-flux density.





$$\alpha(\nu) = \frac{\alpha_0(\nu)}{1 + \frac{\Phi}{\Phi_s(\nu)}}$$

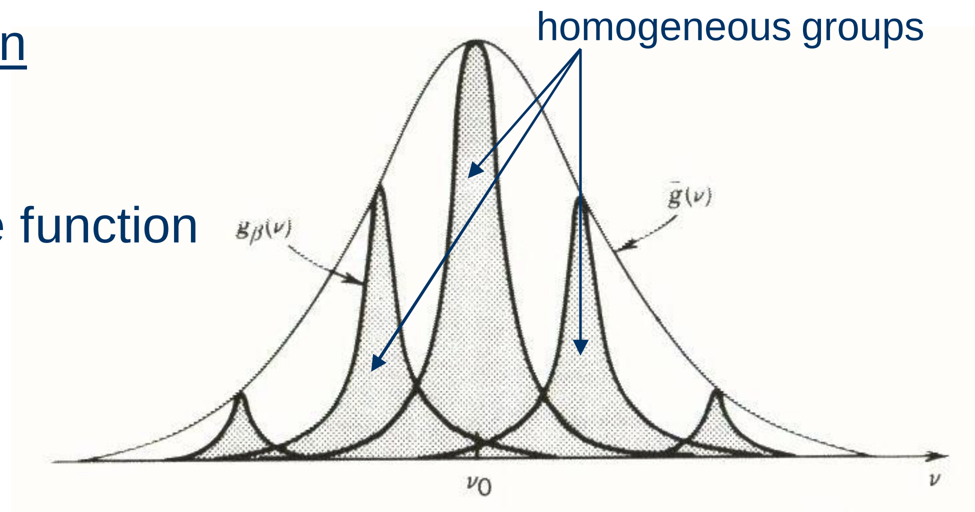
Application: passive Q-switching!

Coherent optical amplifier

<u>Amplifier nonlinearity or gain saturation</u>

Inhomogeneous saturable amplifier

Small signal gain ~ average lineshape function



$$\overline{\gamma_0}(\nu) = N_0 \frac{\lambda^2}{8\pi\tau_{sp}} \overline{g}(\nu), \qquad \frac{\lambda^2}{8\pi\tau_{sp}} = S,$$

$$\overline{g}(\nu) = \left\langle g_\beta(\nu) \right\rangle$$

$$\overline{\gamma}(\nu) = \langle \gamma_\beta(\nu) \rangle, \qquad \gamma_\beta(\nu) = \frac{\gamma_{0\beta}(\nu)}{1 + \dfrac{\Phi}{\Phi_{s\beta}(\nu)}}, \qquad \gamma_{0\beta} = N_0 S\, g_\beta(\nu), \qquad \frac{1}{\Phi_{s\beta}} = S\tau_s\, g_\beta(\nu)$$

Lineshape function of a homogeneous group of β (e.g. particle moving with the same velocity in the direction of the photon beam)

$$\longrightarrow \quad g_\beta(\nu) = \frac{\dfrac{\Delta\nu}{2\pi}}{(\nu - \nu_\beta - \nu_0)^2 + \left(\dfrac{\Delta\nu}{2}\right)^2}$$

Coherent optical amplifier

Amplifier nonlinearity or gain saturation

Inhomogeneous saturable amplifier (cont.)

$$\gamma_\beta(\nu) = N_0 S \frac{\frac{\Delta\nu}{2\pi}}{\left(\nu - \nu_\beta - \nu_0\right)^2 + \left(\frac{\Delta\nu_s}{2}\right)^2}, \quad \Delta\nu_S = \Delta\nu\left(1 + \frac{\Phi}{\Phi_S(\nu_0)}\right)^{1/2},$$

$$\frac{1}{\Phi_s(\nu_0)} = \frac{\lambda^2}{8\pi}\frac{\tau_s}{t_{sp}}\frac{2}{\pi\Delta\nu} = \frac{\lambda^2}{8\pi}\frac{\tau_s}{t_{sp}} g(\nu_0).$$

The β–group has a fraction according to the Maxwell-Boltzmann distribution:

$$p(\nu_\beta) = \frac{1}{\sigma_D (2\pi)^{1/2}} e^{-\frac{\nu_\beta^2}{2\sigma_D^2}}, \quad \Delta\nu_D = 2\sigma_D (ln\,2)^{1/2}, \quad \frac{1}{\sigma_D} = \left(\frac{M}{kT}\right)^{1/2}\frac{c}{\nu_0}.$$

Calculation of the saturated gain: $\boxed{\overline{\gamma}(\nu) = \int_{-\infty}^{\infty} \gamma_\beta(\nu)\, p(\nu_\beta)\, d\nu_\beta}$

Coherent optical amplifier

<u>Amplifier nonlinearity or gain saturation</u>

Inhomogeneous saturable amplifier (cont.)

$$\int_{-\infty}^{\infty} \frac{a\,dx}{a^2+x^2} = \pi$$

Analytic solution only in special cases, e.g. when $\Delta\nu_D >> \Delta\nu_s$, around ν_0 the saturated gain:

$$\nu = \nu_0, \quad p(\nu_\beta = 0) = \frac{1}{\sigma_D (2\pi)^{1/2}}, \quad \bar{\gamma}(\nu_0) = \frac{N_0 S \dfrac{\Delta\nu}{2\pi}}{(2\pi)^{1/2}\sigma_D \left(\dfrac{\Delta\nu_s}{2}\right)} \int_{-\infty}^{\infty} \frac{\dfrac{\Delta\nu_s}{2} d\nu_\beta}{\nu_\beta^2 + \left(\dfrac{\Delta\nu_s}{2}\right)} =$$

$$= \frac{N_0 S}{(2\pi)^{1/2}\sigma_D} \frac{\Delta\nu}{\underset{\Delta\nu\left(1+\frac{\Phi}{\Phi_s(\nu_0)}\right)^{1/2}}{\underset{\uparrow}{\Delta\nu_s}}} = \frac{N_0 S}{(2\pi)^{1/2}\sigma_D} \frac{1}{\left[1+\dfrac{\Phi}{\Phi_s(\nu_0)}\right]^{1/2}} = \frac{\overline{\gamma_0}}{\left[1+\dfrac{\Phi}{\Phi_s(\nu_0)}\right]^{1/2}}$$

$$\boxed{\bar{\gamma}(\nu_0) = \frac{\overline{\gamma_0}}{\left[1+\dfrac{\Phi}{\Phi_s(\nu_0)}\right]^{1/2}}, \quad \bar{\gamma}_0 = N_0 \frac{\lambda^2}{8\pi\, t_{sp}} \frac{1}{\sqrt{2\pi\sigma_D^2}}.}$$

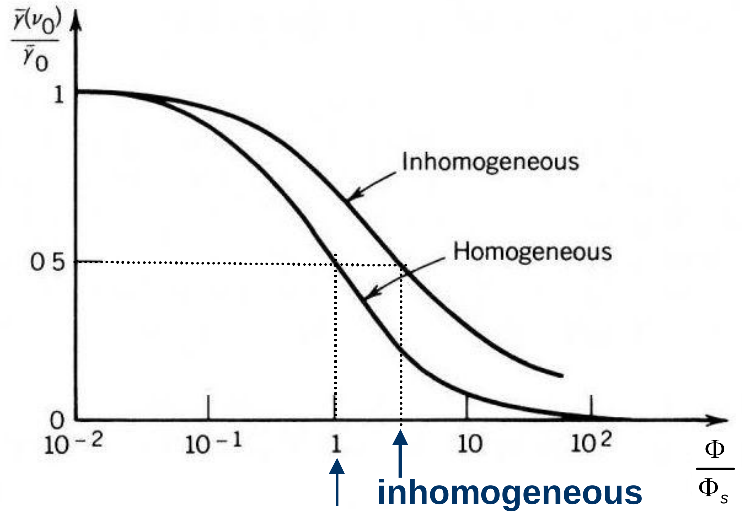
Coherent optical amplifier

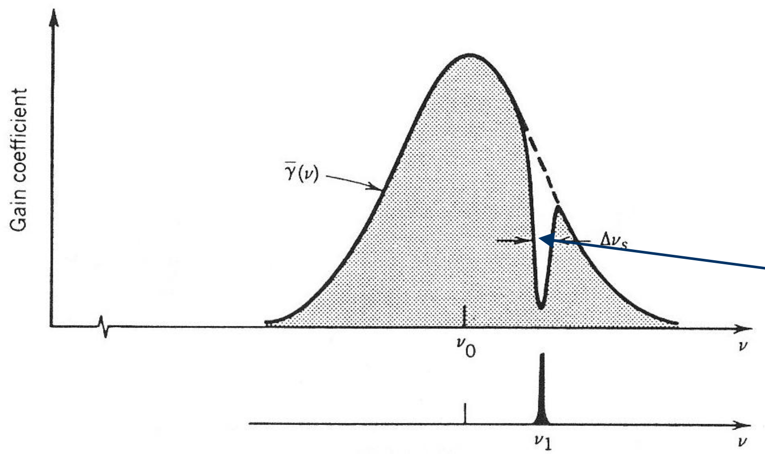
<u>Amplifier nonlinearity or gain saturation</u>

Inhomogeneous saturable amplifier (cont.)

$$\frac{\bar{\gamma}(\nu_0)}{\bar{\gamma}_0}=\frac{1}{2}, \quad 4=\left[1+\frac{\Phi}{\Phi_s}\right]$$

$$\boxed{\Phi=3\Phi_s \qquad \nu=\nu_0}$$





Local saturation by a large flux density photon beam of frequency ν_1

"spectral hole burning"

The width and depth of the hole increases with the flux density.

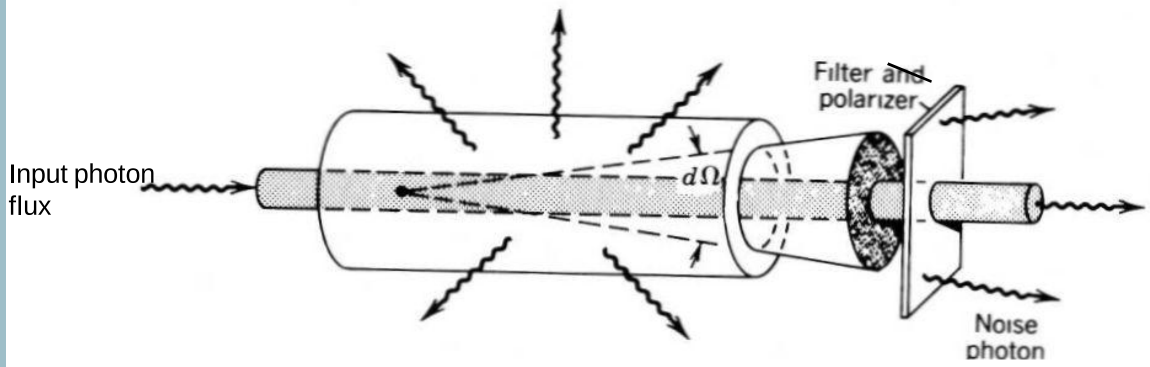
Coherent optical amplifier

Amplifier noise

The amplified spontaneous emission (ASE) noise is broadband, multidirectional, and unpolarized. The probability density (per second) of spontaneous emission in the range $[\nu, \nu + d\nu]$ and in unit volume dV:

$$P_{sp}(\nu)d\nu = \frac{1}{t_{sp}} g(\nu) d\nu, \qquad P_{sp} = \frac{1}{t_{sp}} \int_0^\infty g(\nu) d\nu$$

if N_2 is the atomic density in level 2, the average spontaneously emitted power per unit volume per unit frequency is: $h\nu N_2 P_{sp}(\nu)$. The number of emitted photons in unit length of the unit volume within bandwidth B around ν in solid angle $d\Omega$ with a given polarization:



$$\xi_{sp}(\nu) = N_2 \frac{1}{t_{sp}} g(\nu) B \frac{1}{2} \frac{d\Omega}{4\pi}$$

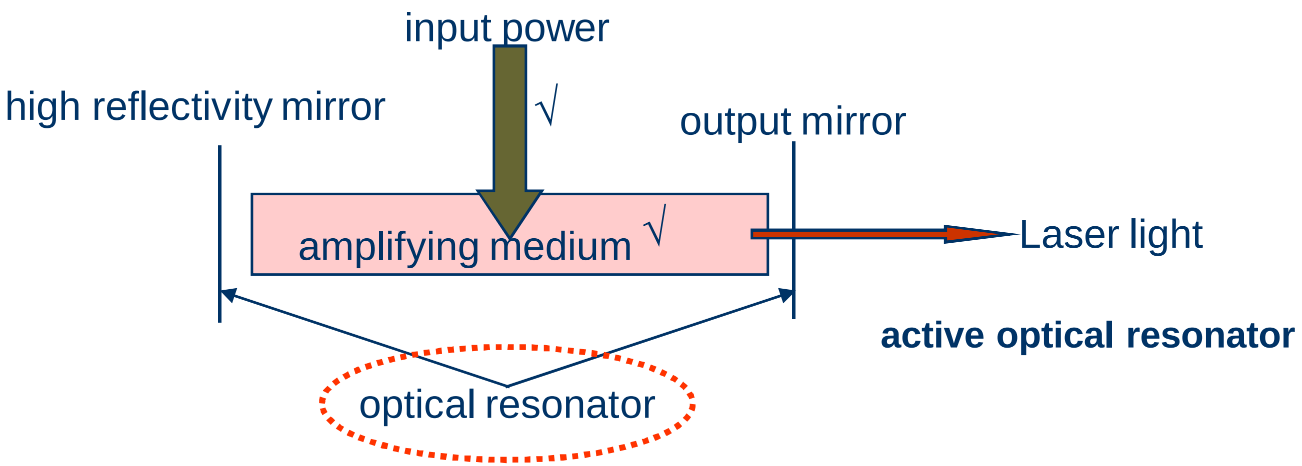
polarization

$$\frac{d\Phi}{dz} = \gamma(\nu)\Phi + \xi_{sp}(\nu)$$

Passive optical resonators

Passive optical resonators

What is necessary for a laser?



Optical resonator – cavity with reflecting walls, provides feedback of the electromagnetic field on discrete frequency, determines spatial distribution of the modes.

Passive optical resonator – no active medium is present.

Passive optical resonators

Characteristics of passive optical resonators

- They are open, feedback only from a narrow solid angle (no side walls and small size mirrors in the longitudinal direction),
- Dimensions >> λ_{laser}, suitable length of the active medium depends on the gain.

Solving Maxwell-equations for the geometry of the optical cavity (solving wave-equation with boundary conditions: the field amplitude is taken to be ”0”) → discrete frequency modes of the electromagnetic fields can be determined. Because of the open resonator instead of $\underline{E}(\underline{r},t)=E_0\underline{U}(\underline{r})\exp(j2\pi\nu t)$ the usual stationary solutions

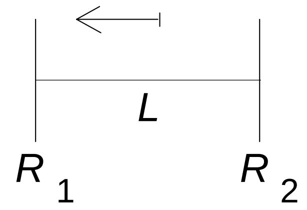
$$\boxed{\underline{E}(\underline{r},t)=E_0\underline{U}(\underline{r})\exp\{(-t/2\tau_r)+j2\pi\nu t\}}$$

modes with exponentially decaying amplitude, τ_r is the resonator or photon lifetime.

Passive optical resonators

<u>One dimensional plane parallel resonator</u> – estimation of the photon lifetime

L is the resonator length, R_1 and R_2 are reflectivity (intensity) of the mirrors, α_r is the loss coefficient in the resonator (unit length), α_s is the scattering coefficient between the mirrors. For one round-trip the intensity changes:



$$e^{-2\alpha_r L} = R_1 R_2 e^{-2\alpha_s L}, \quad \frac{1}{\alpha_r c} = \tau_r, \quad \boxed{\Delta\nu_r = \frac{1}{2\pi\tau_r}}, \quad \alpha_r = \frac{2\pi}{c}\Delta\nu_r.$$

resonator or photon lifetime

While α_s<<1

$$e^{-2\alpha_r L} = R_1 R_2, \quad -2\alpha_r L = \ln R_1 R_2, \quad \alpha_r = \frac{-\ln R_1 R_2}{2L}.$$

E.g., if $R = R_1 = R_2 = 0.98$ and $L = 0.5\ m$

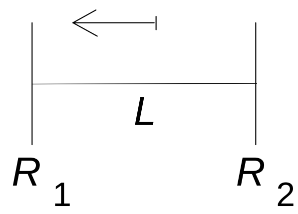
$$\tau_r = \frac{1}{\alpha_r c} = \frac{2L}{-c\ln R_1 R_2} = \frac{2L}{c}\frac{1}{-\ln R_1 R_2} = \frac{t_r}{-\ln R_1 R_2}$$

$$t_r = 3.33\,ns, \quad \tau_r = 82.5\,ns \quad \text{és} \quad \Delta\nu_r \approx 2\,MHz.$$

round-trip time

~ 25 t_r

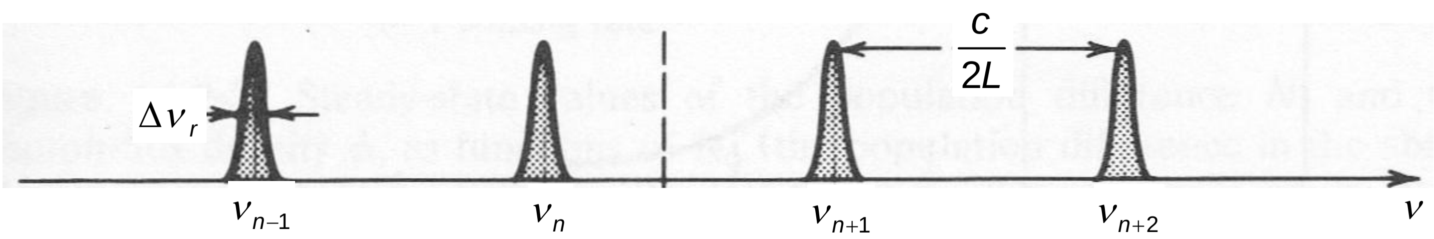
Passive optical resonators



One dimensional plane parallel resonator – modes

From the standing waves condition:

$$L = n\frac{\lambda}{2}, \quad n \text{ pozitive integer}, \quad \nu_n = \frac{c}{\lambda} = n\frac{c}{2L}.$$



Equidistant (c / 2L) Lorentz-type modes with a bandwidth of $\Delta\nu_r$.

If: $R = R_1 = R_2 = 0.98$ and $L = 0.5\ m$

$$\Delta\nu = \nu_{n+1} - \nu_n = \frac{3\cdot 10^8}{2\cdot 0.5} Hz = 300\ MHz, \quad \Delta\nu_r \approx 2MHz.$$

Passive optical resonators

Quality (Q) factor

$$Q \underset{\text{def}}{\equiv} \frac{2\pi \cdot \textit{stored energy}}{\textit{energy loss in one oscillation cycle}} = \frac{2\pi \cdot \nu}{\alpha_r c} = \frac{\nu}{\Delta \nu_r} = 2\pi\tau_r\nu.$$

$$\frac{\alpha_r c}{\nu}$$

Ex.: $R = R_1 = R_2 = 0.98$; $L = 0.5$ m, $\nu = 5 \cdot 10^{14} Hz$ ($0.6\ \mu m$)

$$Q = 2.5 \cdot 10^8$$

Q increases with increasing resonator lifetime, high Q-values can be achieved in a resonator when the bandwidth of the modes is small!

Passive optical resonators

Types of resonators

Mirrors can be rectangular or circular, plane, concave or convex, in a distance of few cm's to few meters. Dimensions of the mirrors are typically few mm's or cm.

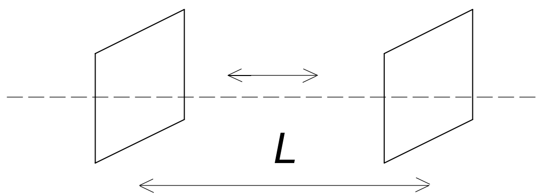
The geometry determines:

- the volume of the modes in the cavity,
- the gain,
- properties of the laser beam such as diameter and divergence.

Passive optical resonators

Types of resonators

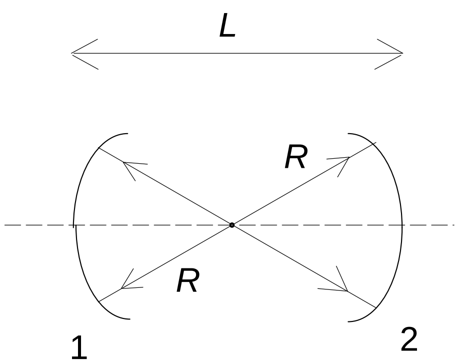
1. Plane parallel (or Fabry-Perot) resonator



Superposition of two plane waves traveling in opposite directions along the cavity axis.

$$L = n\frac{\lambda}{2}, \quad n \text{ pozitive integer}, \quad \boxed{\nu_n = \frac{c}{\lambda} = n\frac{c}{2L}}.$$

2. Concentric or spherical resonator (R is radius)



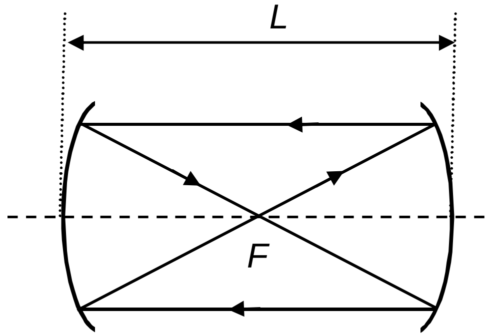
$L = 2R$

Superposition of two oppositely traveling spherical waves. The resonant frequencies are equal with the frequencies of the Fabry-Perot resonator.

Passive optical resonators

<u>Types of resonators</u> (cont.)

3. Confocal resonator (special role)

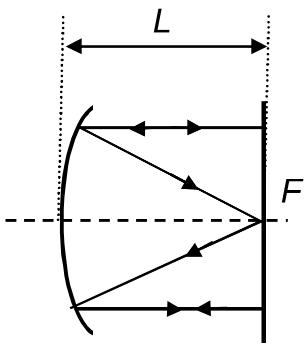


$L = R$, $R_1 = R_2 = R$, $F_1 = F_2 = F$

The modes are not plane or spherical waves and the resonant frequencies have no simple form.

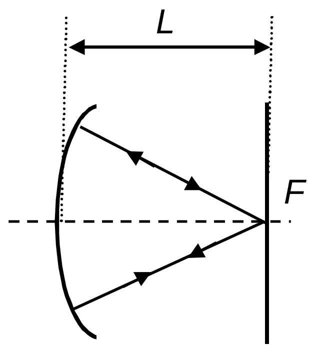
4. Plane and spherical mirror combinations

hemi-confocal (half of 3.)



$$L = \frac{R}{2}$$

hemi-spherical (half of 2.)



$$L = R$$

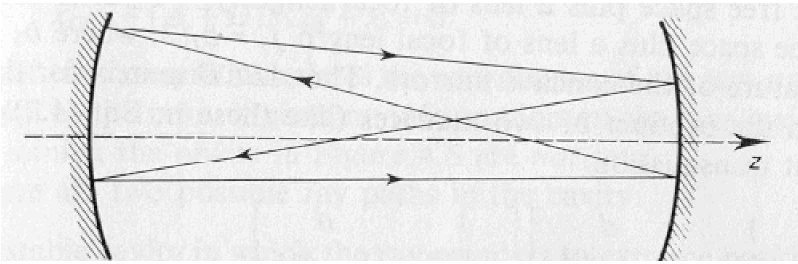
Passive optical resonators

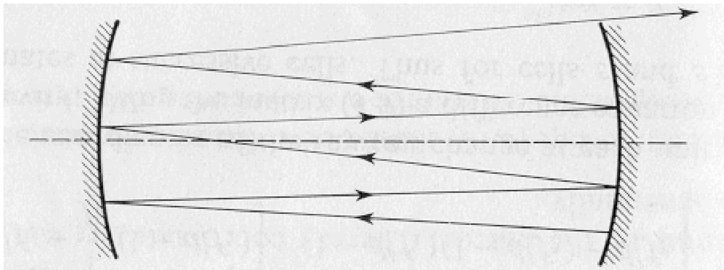
Types of resonators (cont.)

5. General resonator

Two mirrors with optional spherical radius in a distance of L. Task: determination of the spatial distribution, the frequency and the loss of the modes. Two categories:

stable resonator	unstable resonator
rays remain inside, repeated ray-paths	after some round-trip the ray diverges from the cavity





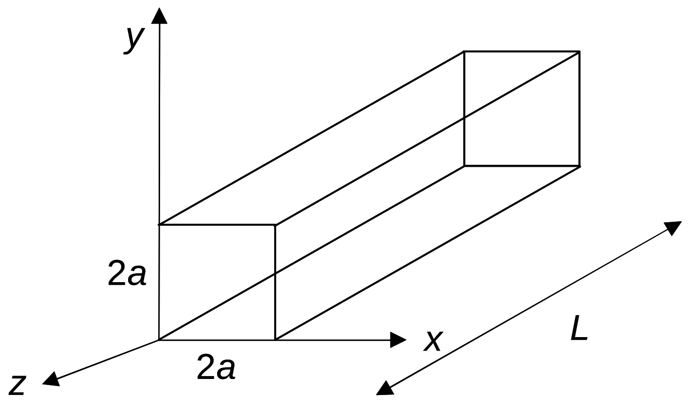
Passive optical resonators

<u>Plane parallel resonator</u> – approximate determination of $\nu_{l,m,n}$ frequencies

$$2a = l\frac{\lambda}{2} \qquad k_x = \frac{2\pi}{\lambda} = \frac{l\pi}{2a} \qquad |\underline{k}| = \frac{2\pi}{\lambda} = \frac{2\pi\nu}{c}$$

$$\lambda = \frac{4a}{l} \qquad k_y = \frac{m\pi}{2a} \qquad k_z = \frac{n\pi}{L}$$

$$|\underline{k}| = \sqrt{k_x^2 + k_y^2 + k_z^2}$$



$$\nu_{l,m,n} = \frac{c|\underline{k}|}{2\pi} = \frac{c}{2}\left[\left(\frac{n}{L}\right)^2 + \left(\frac{m}{2a}\right)^2 + \left(\frac{l}{2a}\right)^2\right]^{1/2}$$

Removing side walls → open resonator, *l, m* << *n* (in practice 0, 1 or 2)

$$\nu_{\underbrace{l,m}_{},n} = \frac{c\,n}{2L}\left[1 + \left(\frac{L}{n}\right)^2 \left\{\left(\frac{m}{2a}\right)^2 + \left(\frac{l}{2a}\right)^2\right\}\right]^{1/2} \cong \frac{c}{2}\left(\frac{n}{L} + \frac{L}{2n}\frac{m^2 + l^2}{4a^2}\right)$$

degeneration!

transverse

longitudinal

indexes

$$\sqrt{1+x} \cong 1 + \frac{1}{2}x, \quad \text{for small } x$$

Passive optical resonators

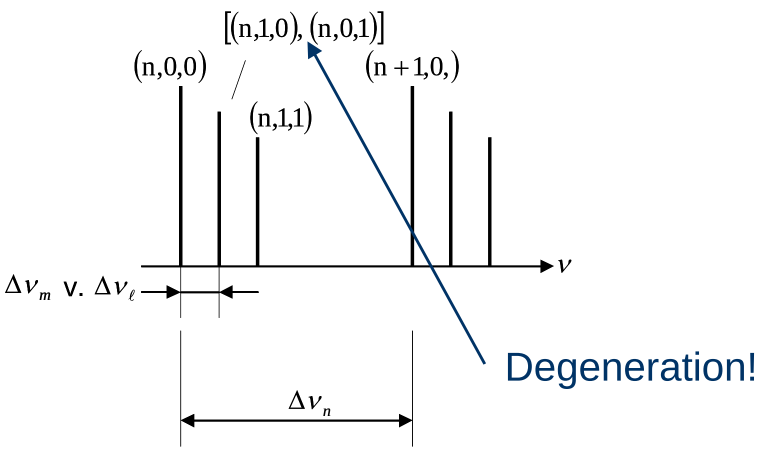
Plane parallel resonator – approximate determination of *l,m,n* frequencies

Distance of two consecutive longitudinal modes: $\boxed{\Delta\nu_n = \nu_{l,m,n+1} - \nu_{l,m,n} = \frac{c}{2L}}$

If $L = 0.5\ m$, $\Delta\nu_n = 3\cdot10^8\ s^{-1} = 300\ MHz.$ Typical order of magnitude: 100 *MHz* ,

Distance of two consecutive transverse modes:

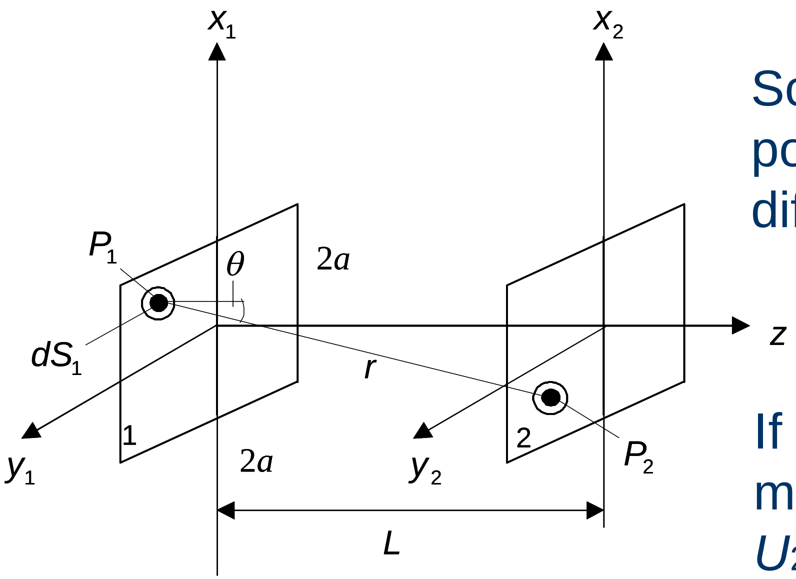
$$\boxed{\Delta\nu_m = \nu_{l,m+1,n} - \nu_{l,m,n} = \frac{cL}{8na^2}\left\{\frac{(m+1)^2 - m^2}{2}\right\} = \frac{cL}{8na^2}\left(m + \frac{1}{2}\right)}$$



Typical order of magnitude : ~ *MHz*

Passive optical resonators

<u>Plane parallel resonator</u> – amplitude distribution of transverse modes and calculation of the loss



Scalar diffraction theory (condition: uniform polarization of the e.m. field). The Kirchhoff diffraction integral

$$U_2(P_2) = -\frac{i}{2\lambda}\int_1 \frac{U_1(P_1)exp(ikr)(1+\cos\theta)}{r}dS_1$$

If U is the amplitude distribution of a resonator mode and the two mirror are identical, $U_1(P_1)$ and $U_2(P_2)$ can differ only with a constant factor.

$$\sigma U(P_2) = -\frac{i}{2\lambda}\int_1 \frac{U(P_1)\exp(ikr)(1+\cos\theta)}{r}dS_1, \quad \sigma = |\sigma|\exp(i\phi),$$

Numerical solution!

$\gamma_d = 1-|\sigma|^2$ diffraction loss and phase shift for a round-trip in the resonator, q is a positive integer
$2\phi = 2\pi \cdot q$

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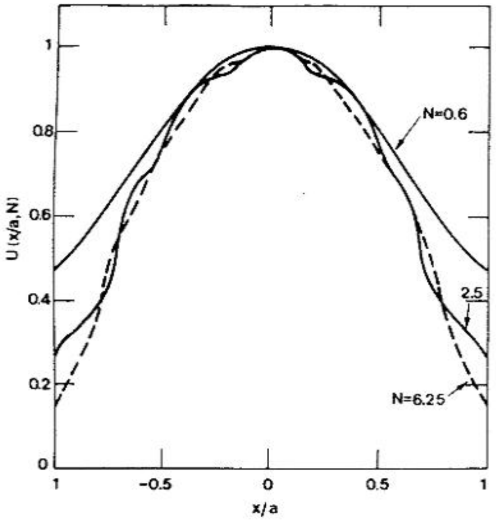
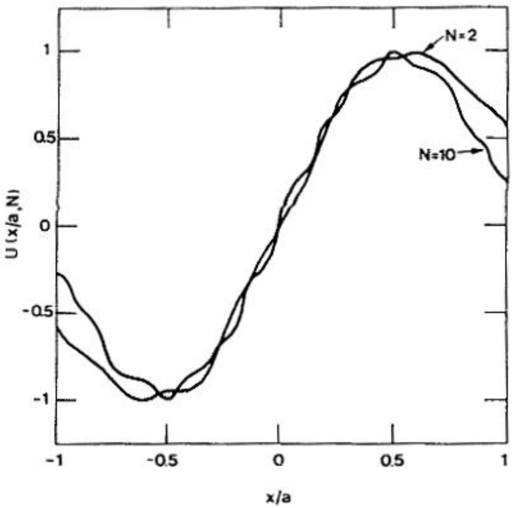
symmetrical mode



asymmetric mode



The parameter N in figures is the Fresnel number, the ratio of the geometrical angle and twice the diffraction angle:

$$N \underbrace{=}_{def.} \frac{\theta_g}{2\theta_d} = \frac{a}{L}\cdot\frac{a}{\lambda} = \frac{a^2}{L\cdot\lambda}, \quad \theta_d = \beta\frac{\lambda}{2a} \quad \beta \approx 1, \quad \theta_g = \frac{a}{L}.$$

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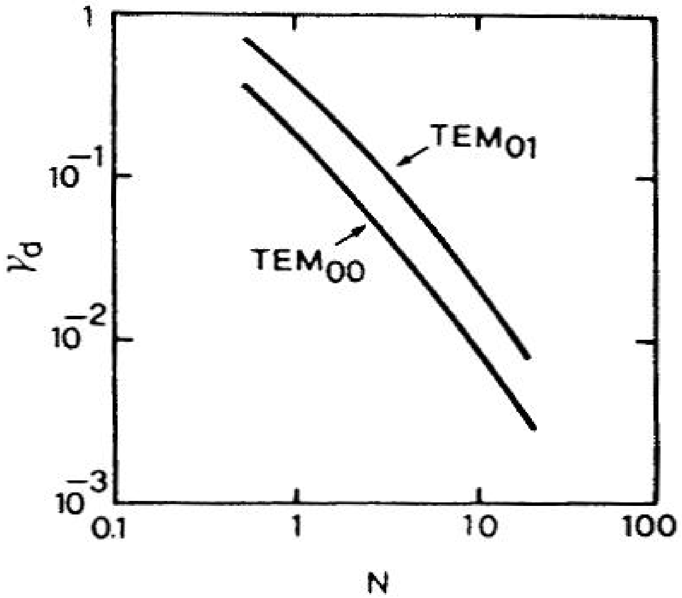
γ_d depends on *N* and the transverse mode indices *l and m,* and independent on *n*:



Loss of TEM_{00} and TEM_{01} as a function of the Fresnel number

The notation of the transverse modes is TEM_{ml}

transverse electromagnetic mode