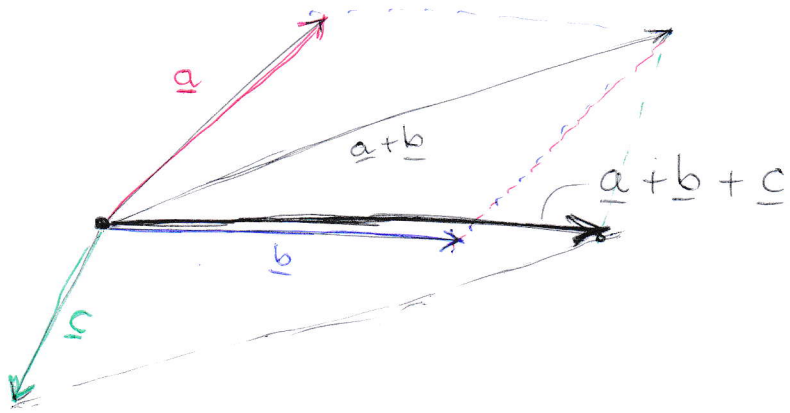
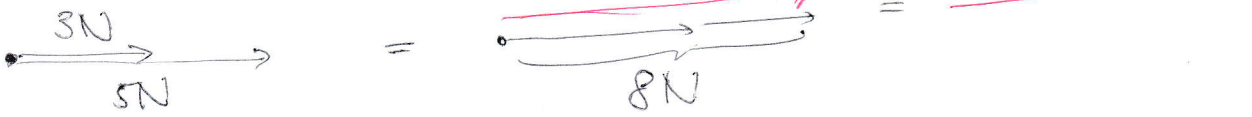
Feladatok megoldása 2

F1

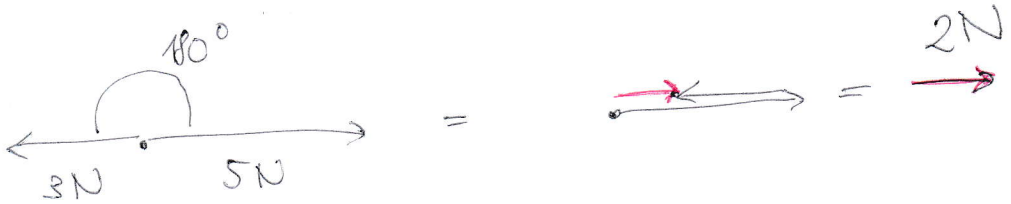


F2

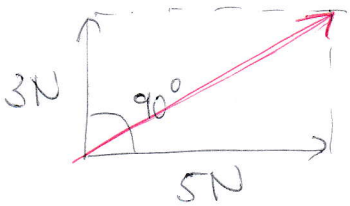
a.)



b.)



c.)



Pitagorasz tétel

$$F_e = \sqrt{3^2 + 5^2} = \sqrt{34}\,N =$$

$$= \underline{5.83\,N}$$

d.)

Cosinus-tétel!

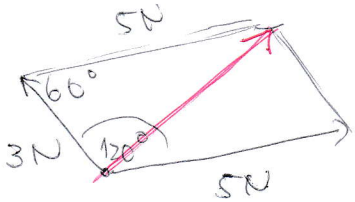
$$F_e^2 = 3^2 + 5^2 - 2 \cdot 3 \cdot 5 \cos 120^\circ =$$

$$= 34 - 30 \cdot \cos 120^\circ = 34 + 15$$

$$= 49\,N^2$$

$$L_D \quad F_e = \sqrt{49}\,N = 7N$$

e.)

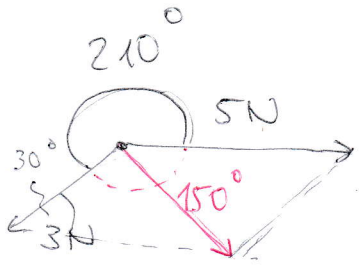


$$F_e^2 = 3^2 + 5^2 - 2\cdot 3\cdot 5\cdot \cos 60^\circ =$$

$$= 34 - 15 = 19\,N^2$$

$$F_e = \sqrt{19}\,N = 4{,}36\,N$$

f.)



$$F_e^2 = 3^2 + 5^2 - 2\cdot 3\cdot 5\cdot \cos 30^\circ =$$

$$= 34 - 30\cdot \frac{\sqrt{3}}{2} = 8{,}02\,N^2$$

$$F_e = 2{,}83\,N$$

F3 a.) $\underline{a} = (1, 2, 3)$

$\underline{b} = (0, 4, 6)$

$\underline{c} = (-5, 2, 1)$

a.) $\underline{a} + \underline{b} + \underline{c} = (1+0-5,\ 2+4+2,\ 3+6+1)$

$= (-4, 8, 10)$

b.) $\underline{a} - \underline{b} + \underline{c} = (1-0-5,\ 2-4+2,\ 3-6+1)$

$= (-4, 0, -2)$

c.) $3\underline{a} + 2\underline{b} - 5\underline{c} = (3\cdot 1 + 2\cdot 0 + 5\cdot 5,$
$3\cdot 2 + 2\cdot 4 - 5\cdot 2,$
$+3\cdot 3 + 2\cdot 6 - 5\cdot 1)$

$= (28, 4, 16)$

d.) $\underline{a}\cdot\underline{c} = 1\cdot(-5) + 2\cdot 2 + 3\cdot 1 = 2$

e.) $(\underline{a}\cdot\underline{b})\underline{c} = (1\cdot 0 + 2\cdot 4 + 3\cdot 6)\cdot \underline{c} = 26\,\underline{c} =$

$= (-130, 52, 26)$

f.) $\underline{a}\cdot(\underline{b}\cdot\underline{c}) = \underline{a}\cdot(0\cdot(-5) + 4\cdot 2 + 6\cdot 1) = 14\,\underline{a} = (14, 28, 42)$

fontos észrevétel: $\boxed{(\underline{a}\cdot\underline{b})\cdot\underline{c} \neq \underline{a}(\underline{b}\cdot\underline{c})}$

F4 Bezárt szög skalárszorzatból.

$\underline{a}\cdot\underline{b} = |\underline{a}|\cdot|\underline{b}|\cdot\cos\varphi$

ki tudom számolni koordinátákkal

$\hookrightarrow \cos\varphi = \dfrac{\underline{a}\cdot\underline{b}}{|\underline{a}|\cdot|\underline{b}|}$

ehhez

$|\underline{a}| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$

$|\underline{b}| = \sqrt{0^2+4^2+6^2} = \sqrt{52}$

$|\underline{c}| = \sqrt{5^2+2^2+1^2} = \sqrt{30}$

a.) $\underline{a}\cdot\underline{b} = 1\cdot 0 + 2\cdot 4 + 3\cdot 6 = 26 \rightsquigarrow \cos\varphi = \dfrac{26}{\sqrt{14}\cdot\sqrt{52}} = 0.964$

$\boxed{\varphi = 15{,}5^\circ}$

b.) $\underline{a}\cdot\underline{c} = 1\cdot(-5) + 2\cdot 2 + 3\cdot 1 = 2 \rightsquigarrow \cos\varphi = \dfrac{2}{\sqrt{14}\,\sqrt{30}} = 0.0976$

$\varphi = 84{,}4^\circ$

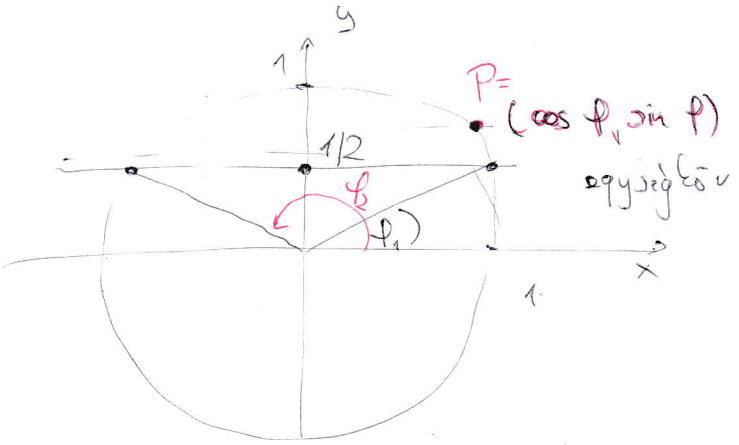
c.) $\underline{b} \cdot \underline{c} = 0 \cdot (-5) + 4 \cdot 2 + 6 \cdot 1 = 14$

$$\cos\varphi = \frac{14}{\sqrt{52} \cdot \sqrt{30}} = 0.354$$

$$\varphi = 69.2°$$

F5

$\sin x = 1/2$



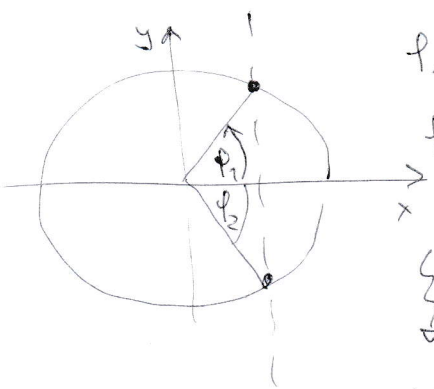
számológép:
(radián!)
Valamit visszaad.
↳ figyelni kell!

$\varphi_1 = \frac{\pi}{6} \approx 0.524$

$\varphi_2 = \frac{5}{6}\pi \approx 2.618$

↳ $x = \begin{cases} \frac{\pi}{6} + n \cdot 2\pi \\ \frac{5}{6}\pi + n \cdot 2\pi \end{cases}$

$\cos x = 1/2$



$\varphi_1 = \frac{\pi}{3} = 1.047$

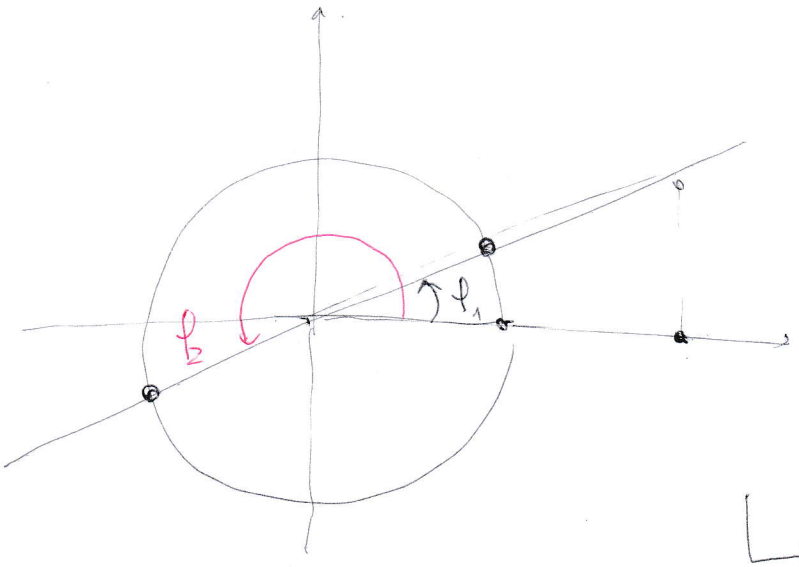
$\varphi_2 = -\frac{\pi}{3}$

$\Downarrow$

$x = \pm \frac{\pi}{3} + n \cdot 2\pi$

tömörített jelölés!

$\tan x = \frac{1}{2}$ $\left(\tan x = \frac{\sin x}{\cos x}\right)$

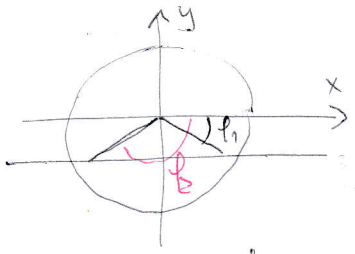


$\varphi_1 = 0.464$

$\varphi_2 = \varphi_1 + \pi$

$\hookrightarrow x = 0.464 + n \cdot \pi$

$\sin x = -\frac{1}{2}$



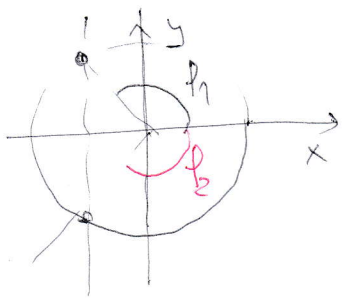
$\varphi_1 = -\frac{\pi}{6}$

$\varphi_2 = -\frac{5}{6}\pi$

Vigyázat, "negatív" szöget?

$x = \begin{cases} -\frac{\pi}{6} + n \cdot 2\pi \\ -\frac{5}{6}\pi + n \cdot 2\pi \end{cases}$

$\cos x = -\frac{1}{2}$



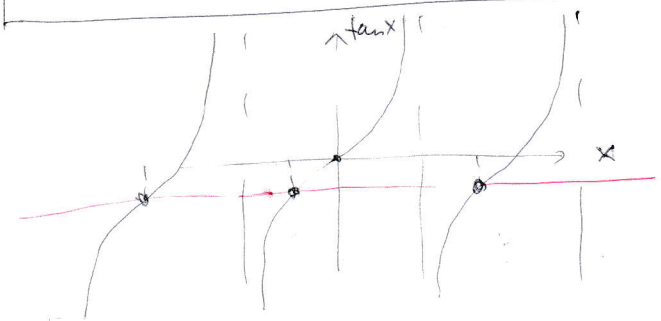
$\varphi_1 = \frac{2}{3}\pi$

$\varphi_2 = -\frac{2}{3}\pi$

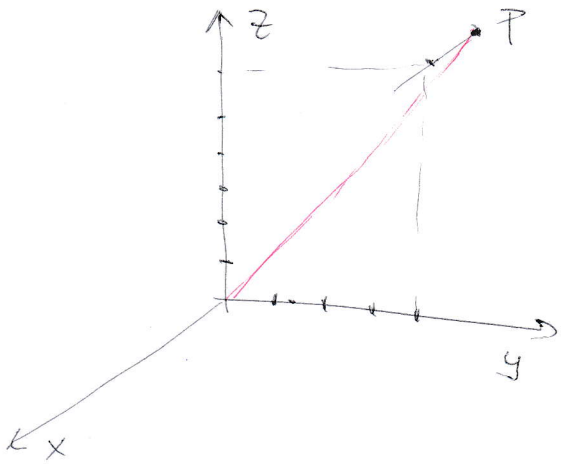
$\hookrightarrow x = \pm\frac{2}{3}\pi + n \cdot 2\pi$

$\tan x = -\frac{1}{2}$

$\boxed{x = -0.464 + n \cdot \pi}$



F6



gömbi

r, ϑ, φ

$r = \sqrt{x^2+y^2+z^2} = \sqrt{53}$

$\cos\vartheta = \frac{z}{r} = \frac{z}{\sqrt{x^2+y^2+z^2}} = 0.824$

$\vartheta = 0.602 = 34,5^\circ$

↑ rad

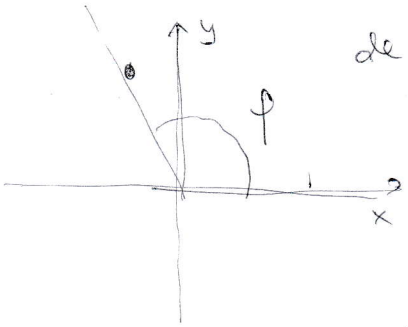
φ:

$\tan\varphi = \frac{y}{x} = -4$ de?

$\varphi = ?$

Vigyázat

(fentről nézve)



de arctan $\pm\frac{\pi}{2}$ köré lépez

$\arctan(-4) = -1.326$

$\varphi = \arctan(-4) + \pi$

$= 1.816 = \underline{104^\circ}$

Henger: (ρ, φ, z)

$\rho = \sqrt{x^2+y^2} = \sqrt{17}$

φ: ugyanaz, mint gömbben

$\hookrightarrow_D \varphi = 104^\circ$

z: ugyanaz, mint derékszögűben!